Working Paper Series —— 3/2021

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Czech National Bank —— Working Paper Series —— 3/2021

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Michal Franta, Jan Libich*

Abstract

We put forward a novel macro-financial empirical modelling framework that can examine the tails of distributions of macroeconomic variables and the implied risks. It does so without quantile regression, also allowing for non-normal distributions. Besides methodological innovations, the framework offers a number of relevant insights into the effects of monetary and macroprudential policy on downside macroeconomic risk. This is both from the short-run perspective and from the long-run perspective, which has been remained unexamined in the existing Macro-at-Risk literature. In particular, we estimate the conditional and unconditional US output growth distribution and investigate the evolution of its first four moments. The short-run analysis finds that monetary policy and financial shocks render the conditional output growth distribution asymmetric, and affect downside risk over and above their impact on the conditional mean that policymakers routinely focus on. The long-run analysis indicates, among other things, that US output growth left-tail risk showed a general downward trend in the two decades preceding the Global Financial Crisis, but has started rising in recent years. Our examination strongly points to post-2008 unconventional monetary policies (quantitative easing) as a potential source of elevated long-run downside tail risk.

Abstrakt

V článku je vyvinut nový makrofinanční modelový rámec, který umožňuje zkoumání okrajů distribuce makroekonomických veličin a implikovaných rizik. Činí tak bez použití kvantilových regresí a s nenormálními distribucemi. Mimo metodologického příspěvku nabízí tento modelový rámec několik nových zjištění ohledně vlivu měnové a makroobezřetnostní politiky na makroekonomický risk, a to nejen z krátkodobé perspektivy, ale i z dlouhodobé perspektivy, která je v tzv. Macro-at-Risk literatuře dosud opomíjena. Konkrétně tedy odhadujeme podmíněnou a nepodmíněnou distribuci růstu HDP ve Spojených státech a zkoumáme vývoj jejích prvních čtyř momentů. Analýza krátkodobých změn ukazuje, že měnověpolitické a finanční šoky vychylují distribuci růstu HDP směrem k asymetrickému tvaru a ovlivňují riziko velmi nízkého růstu více, než ovlivňují průměrný podmíněný růst. Na ten se však hospodářské politiky obvykle zaměřují, a proto mohou dojít k chybným krokům. Analýza dlouhodobých změn ukazuje mimo jiné, že dvě dekády před globální finanční krizí riziko velmi nízkého růstu trendově klesalo, ale v posledních několika letech se tento pokles změnil v nárůst. Náš výzkum ukazuje na nekonvenční nástroje měnové politiky aplikované po roce 2008 jako na potenciální zdroj tohoto zvýšeného rizika dlouhodobě velmi nízkého ekonomického růstu.

JEL Codes: C53, C54, E32.

Keywords: Downside tail risk, growth-at-risk, macroeconomic policy, macro-financial modeling, non-normal distribution, threshold VAR, US output growth.

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We would like to thank Christian Matthes, Martin Kábrt and seminar participants at the Czech National Bank and at the IAAE 2021 for their comments. The views expressed are those of the authors and do not necessarily represent those of the Czech National Bank.

1. Introduction

Mark Twain once argued that "if you hold a cat by the tail you learn things you cannot learn any other way." This paper shows that Twain's argument can be extended from cats to economies. We propose a novel macro-financial empirical modelling framework to broaden and deepen our understanding of macroeconomic risks by focusing on distribution tails, and by empirically examining the distributions' higher moments. The analysis can thus provide answers to relevant policy questions and enhance social welfare by identifying appropriate (risk-reducing) policy actions and frameworks. This is highly desirable; recent years have provided a sobering reminder of the many layers of uncertainty and peril present in people's lives.

Historically, researchers have focused on the mean (the first moment) when modeling macroeconomic relationships. The period described as the Great Moderation brought the issue of volatility (the second moment) to the fore, but it was generally taken to support the earlier focus on the mean as a useful tool for policymakers. However, the sentiment started to change with the 2007-2009 Global Financial Crisis (GFC). Since then, academic economists and policymakers have attempted to understand the likelihood and dynamics of extreme scenarios that occur at the tails, far from the mean of macroeconomic variables (e.g. Cecchetti, 2008). Discussions of the associated risk have been gaining in prominence over time, especially in regards to downside (left-tail) risk.

A rapidly growing body of Macro-at-Risk research has gone beyond mean and volatility, empirically investigating specific distribution quantiles in order to shed some light on downside risks to economic performance.[1] We attempt to advance this line of work in several directions, including a careful examination of skewness (the third moment) and kurtosis (the fourth moment). The contribution of the paper can be divided into two broad areas: econometric methodology and macroeconomic policy.

In terms of the methodological contributions, we expand the conventional macro-financial empirical modelling framework towards simulating full predictive distributions. We do not rely on a nonparametric approach based on quantile regressions used in the majority of Macro-at-Risk studies. Instead, our framework makes progress within the newly emerging stream of this literature that simulates predictive distributions using fully parametric models.[2]

We enrich this literature by employing a nonlinear model that permits asymmetric predictive distributions with fat tails. This means that we can formally explore higher moments of the distributions. Importantly, the distributions are allowed to be multimodal, which the recent literature shows to be important (see e.g. Adrian et al., 2021).[3] Furthermore, our approach allows us to capture

[1] The prominent topic in the Macro-at-Risk literature has been Growth-at-Risk, focusing on lower percentiles (usually the 5th or the 10th) of the conditional output growth distribution several quarters ahead (see Adrian et al., 2019). Studies in the Macro-at-Risk literature have also examined inflation (Banerjee et al., 2020) and capital flows (Eguren-Martin et al., 2020).

[2] This body of research has focused on Growth-at-Risk and employed Bayesian vector autoregressions with stochastic volatility (Carriero et al., 2020), Markov-switching models (Caldara et al., 2020), GARCH-type models (Brownlees and Souza, 2021), and skewed t-distributions with time-varying parameters (Delle Monache et al., 2020, and Plagborg-Møller et al., 2020). All these studies have substantiated the use of parametric approaches owing to their out-of-sample predictive ability for tails of the output growth distribution.

[3] In line with our arguments for approaches based on predictive distributions, Geweke and Keane (2007) point out that quantile regressions provide only a finite number of quantile estimates, whereas the research question can be related to the entire distribution. Furthermore, there is no guarantee that the estimated quantiles follow the ordering

nonlinearities between financial conditions and the real economy, as well as the behavior of the system at longer horizons. One of the advantages is therefore that macroeconomic tail risk can be examined not just from a short-run perspective, but also from a long-run perspective.

The long-run perspective is absent from the literature; we are aware of only one other study that has made a step in this direction so far. In particular, Delle Monache et al. (2020) model explicitly the trend and cyclical deviations in skewed t-distribution parameters (location, scale, shape and heavy-tailedness). In contrast to their purely statistical approach, we stay within a framework of a standard (reduced-form) multivariate representation of the macroeconomic relationships. This allows us to provide an economic narrative of the changes in long-run macroeconomic tail risk, as well as to highlight the policy implications.

A separate methodological contribution of this paper is an improvement of the estimation efficiency within the class of simple nonlinear models, specifically the Gibbs sampler when the conditional posteriors of some model parameters are not analytically tractable.

As for the macroeconomic policy contributions, our framework enables us to understand macroeconomic tail risk and to answer the relevant questions central bankers and regulators require to make policy decisions. Existing Growth-at-Risk studies examine questions such as: "How low could a country's output growth fall in one year's time in the worst 1%, 5% or 10% of adverse economic scenarios?" While this is valuable, we extend the scope of inquiry in two directions. The first relates to our focus on the distribution's higher moments; the second relates to the inclusion of the long-run perspective.

In terms of the former, we demonstrate that an assessment of output growth tail risks based on particular distribution quantiles will unlikely capture the full picture, and that an adequate policy response requires understanding changes to the entire distribution. The analysis thus enables us to answer additional policy-relevant questions in relation to downside risk. One example linked to the third moment is the following: "Does a higher probability of an economic upturn compensate for a higher probability of an economic downturn?" Using the framework we can answer such questions, as it formally models the changes in the distribution's asymmetry over time. An example related to the fourth moment is: "Has the probability of severe recessions and major economic crises increased or decreased over time?" This requires a better understanding of the distribution's fat-tailedness that can also be obtained within the framework proposed here.

In relation to lengthening the time horizon, our analysis provides answers to all the above questions both from the short- and long-run perspective. The short-run perspective, which has been the focus of the literature, conditions on developments in a given quarter/year, and thus assesses the tail risk a few quarters ahead. In contrast, the novel long-run perspective provides unconditional estimates, so it offers results about average tail risk; over and above the short-term risk fluctuations. It can therefore be used to assess the tail risk arising from longer-term factors such as the structure of the economy, economic trends and alternative policy frameworks. For example, the long-run perspective can identify potential differences between conventional and unconventional monetary

from lower to higher quantiles, implying a potential loss of efficiency in a finite sample due to the so-called quantile crossing.

policy, and contribute to the literature on regime switching in the US economy by linking it to macroeconomic tail risks.

In order to report both the short-run and long-run estimates of US Growth-at-Risk, as well as conditional and unconditional output growth distribution moments, we estimate predictive distributions using 30-year data windows from 1964Q2-1994Q1 to 1989Q2-2019Q1. An important advantage is that both the short-run and the long-run perspective draw on the same model, so they are mutually consistent. This means that the analysis can discover valuable information regarding the relationship between short- and long-run tail risk.

We examine the distributional effects of two types of structural shocks. One is financial in nature, namely a change in the excess bond premium, which can also be interpreted as a change in the macroprudential policy stance. The other is a monetary policy shock.[4] We then investigate trends in downside output growth tail risk and the role of policy frameworks by focusing on changes in model parameters and policy shocks.

Let us summarize the main findings and key policy insights. The short-run perspective demonstrates that downside US output growth tail risk is strongly influenced by economic and policy shocks, and the influence is different from the conditional means. This is in line with the results reported in the related literature (including the papers cited in the previous footnote). We add to these findings by showing the effects of distribution moments. While neither of the shocks has a statistically significant short-run effect on kurtosis, i.e. fat-tailedness, all have substantial and diverse short-run effects on skewness, i.e. asymmetry. In particular, we find that an unexpected monetary policy tightening makes the output growth distribution asymmetric in the short-term, with a marked increase in the probability of very low growth. We further show that the credit spread shock influences the distribution's symmetry in the same direction (towards a longer left tail) for both an unexpected easing of financial conditions (a fall in the risk premium) and an unexpected tightening (a rise in the risk premium). Financial shocks can thus lead to a rise in short-term macroeconomic downside risk, implying an important warning to macroprudential policymakers.

In terms of the long-run perspective, the key finding is that post-GFC unconventional monetary policies, namely QE tapering since 2014, have led to a sizeable increase in output growth risk. Putting this into a broader context, we show that between the 1960s and the GFC long-term downside risk to US output growth followed a decreasing trend. There was a fall in the unconditional variance of output growth (known as the Great Moderation, e.g. Stock and Watson, 2003), which more than offset the well-documented gradual decline in the steady state output growth rate (see Antolin-Diaz et al., 2017). Crucially, we demonstrate that this offsetting effect has no longer been operational in the post-GFC period, which is why long-run downside risk has been on the rise since 2014. We show that developments in the unconditional skewness of output growth are an important factor in this period, and uncover changes in short-term macroeconomic dynamics and the policy shock structure that underlie this. Our analysis is capable of linking both of these changes to post-2008 unconventional monetary policy measures.

[4] This aspect of our analysis contributes to the literature on the impact of economic characteristics and shocks on Growth-at-Risk. The studies have focused on financial variables (Adrian et al., 2019), monetary policy and financial shocks (Duprey and Ueberfeldt, 2020, Loria et al., 2019, Jung and Lee, 2019, Kim et al., 2019), as well as macroprudential policy shocks and indicators (Franta and Gambacorta, 2020, Aikman et al., 2019).

The conclusions section looks at the several implications of and lessons from our short- and long-run findings for macroeconomic modelers, as well as for policymakers. Given that improved empirical analysis together with superior policy actions can enhance social welfare substantially, learning from these lessons is paramount.

2. Empirical Framework

This section describes the two setups employed in our analysis. We start with the empirical macro-financial model used to estimate the entire conditional and unconditional distributions and construct tail risk measures. Then a model for the conditional moments based on the local projections is laid out.

2.1 Model for Simulation of Predictive Distributions

Our approach to the estimation of output growth distributions is based on the simulation of predictive distributions.[5] We use a mean-adjusted threshold vector autoregression with the threshold variable potentially dependent on (the lags of) all endogenous variables. The regime switch is thus linked explicitly to the macroeconomic variables, allowing for the economy to be path-dependent. The model can capture changes in economic relationships, changes in the steady states of endogenous variables, as well as changes in shock volatilities. Importantly, the model also enables nonlinearities between the financial sector and the real economy (Balke, 2000).

An additional benefit is that the parameters of the predictive distribution of output growth (the mean, variance, skewness and kurtosis) can be directly linked to macroeconomic entities. These include regime-specific steady states and regime-specific shock volatilities together with the regime probabilities dependent on macroeconomic variables. Such direct economic interpretation is unavailable in the statistical approaches based on the time-varying Skew-t specification for GDP growth (Delle Monache et al., 2020, and Plagborg-Møller et al., 2020).

The vector autoregressions (VARs) commonly used to estimate predictive distributions allow for stochastic volatility (they were introduced by Cogley et al., 2005, and in the context of Growth-at-Risk employed by Carriero et al., 2020). In order to be able to examine long-run risk, our framework requires bounded variance. Volatility is therefore allowed to change only across the regimes, but not within each regime. The assumption of constant model parameters is addressed (to some extent) by the exploration of parameter changes over different estimation windows.

The model is formulated as follows. For some regime r we have

$$y_t = A_1^{(r)} y_{t-1} + \cdots + A_p^{(r)} y_{t-p} + F^{(r)} - A_1^{(r)} F^{(r)} \cdots - A_p^{(r)} F^{(r)} + \varepsilon_t^{(r)}, \tag{1}$$

where the $n \times 1$ vector y_t denotes the vector of endogenous variables. The coefficient matrices at lag l, denoted $A_l^{(r)}$, are of dimension $n \times n$. The $n \times 1$ vector $F^{(r)}$ contains the unconditional means of endogenous variables. Finally, the vector of error terms $\varepsilon_t^{(r)}$ is distributed independently and normally with a zero mean and a covariance matrix $\Sigma^{(r)}$, i.e. $\varepsilon_t^{(r)} \sim N(0, \Sigma^{(r)})$.

[5] For stationary time series with the Markovian property the conditional predictive distribution fully characterizes the probabilistic aspect of the time series.

Two regimes are considered. They are determined by the threshold variable y_t^{TR} and the threshold value R. The threshold variable is defined as a weighted average of smoothed endogenous variables:

$$y_t^{TR} \equiv \sum_{i=1}^{n} w_i MA^q(y_{it}), \tag{2}$$

where the weights w_i are estimated. The operator $MA^q(\cdot)$ denotes the moving average of the last q observations, whereby we set $q = 4$. A smoothing of the threshold variable components allows for a certain persistence of the regimes. It reflects the commonly held view that a single shock should not generally lead to a regime change, i.e. regime changes are better described by gradual underlying processes (for more justification see, for example, Balke, 2000).

The threshold value R is also estimated. Regimes 1 and 2 are defined for periods when $y_t^{TR} < R$ and $y_t^{TR} \geq R$ respectively. In order for the threshold and weights to be uniquely determined we impose $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. The distinct regimes can potentially capture a number of important features. First, they can account for different steady states, e.g. a low-interest rate (liquidity-trap) regime versus normal times. Second, the regimes can conceivably encapsulate different shock volatilities (e.g. low vs. high volatility). Third, the regimes may pick up different policy behavior (e.g. Taylor-type conventional monetary policy vs. a quantitative-easing type unconventional monetary policy). Finally, the regimes can capture different short-term dynamics, e.g. normal times in the financial markets vs. turbulent times (featuring feedback loops between the financial markets and the real economy).

Following Villani (2009), the standard VAR is reformulated as in (1) to work directly with parameters which can be interpreted as steady states. For a stationary process it follows from (1) that

$$E(y) = F^{(r)}. \tag{3}$$

Finally, we assume that our model describes a stable non-oscillating system, i.e. explosive and oscillating patterns of macroeconomic variables are not considered when the model parameters are sampled and predictive distributions are simulated. The stability of the system is important if the long-run output growth distribution and tail risk are to be examined. Appendix A.3 provides further details.

2.2 Model for Conditional Distribution Moments

The analysis of the conditional distribution moments is built on the following linear regression, estimated by the OLS:

$$\mu_{t+h+4|t+h}^{(m)} = \alpha_h^{(m)} + \beta_h^{(m)} Shock_t + \gamma_h^{(m)} \mu_{t+4|t}^{(m)} + \varepsilon_t^{(m)}, \qquad h = 0, \dots, H \tag{4}$$

where $\mu_{t+h+4|t+h}^{(m)}$ stands for the mean $(m = 1)$, variance $(m = 2)$, skewness $(m = 3)$ and kurtosis $(m = 4)$ in period $t + h + 4$. These moments are estimated in the first stage by employing the model in (1) and using the data available in period $t+h$ (i.e. conditional on the information set Ω_{t+h}). The confidence intervals for the model parameters are bootstrapped; the procedure is described in Appendix B.3.

The variable $Shock_t$ denotes the exogenous structural disturbance.[6] Due to the difficulties involved in identifying structural shock in small-scale models, we do not attempt to identify structural shocks in our threshold VAR model. We proceed in a more robust fashion by employing various shock series produced in the literature. This is done in one of three distinct ways. The first is a narrative identification approach (monetary policy shocks). The second is high frequency identification (monetary policy shocks used in a robustness check). The third uses the timing restrictions imposed on indices derived from micro-level data (excess bond premium shocks).[7]

Note that the linear regression in (4) is reminiscent of the local projections introduced in Jordà (2005), and often applied in the analysis of the effect of shocks on distribution quantiles. Local projections represent a way of estimating the impulse responses if a measure of exogenous variation is available. This is often superior to estimating the full multivariate system and identifying structural shocks. Moreover, local projections allow for a simple examination of nonlinear specifications and the asymmetric effects of the shock, both of which are important for our analysis. For example, variance could be affected similarly by positive and negative shocks; both types of shocks could increase volatility. Thus a linear specification could provide insignificant results, whereas a quadratic specification (or a specification with shocks separated according to the direction of the shock) would provide valuable insights. That is why log specifications, quadratic specifications, and specifications treating positive and negative shocks separately are all examined as robustness checks in Appendices C.1 and C.2.[8]

A further motivation for our focus on distribution moments is that the Growth-at-Risk literature usually studies the effect of a shock on a particular quantile. Alternatively, a skewed t-distribution is fitted to the estimated quantile responses and/or the responses of various quantiles are compared. We focus directly on the effects of the distribution moments, and are thus able to interpret changes in risk from the perspective relevant to the policymaker. As an example, consider a frequently-analyzed monetary policy easing scenario (e.g. Duprey and Ueberfeldt, 2020, Loria et al., 2019, Jung and Lee, 2019). Research studies commonly find that looser monetary conditions shift the left tail of the output growth distribution leftward, in a way that exceeds the shift of the mean. This could be a consequence of several distinct factors: increased conditional variance, a more negative conditional skewness, or fatter tails of the conditional output growth distribution. Finding out which

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[6] The short-run effects of the structural shocks, $\beta_h^{(m)}$, are assumed to be constant over time. Let us however stress that this is not necessarily inconsistent with our approach involving predictive distributions based on estimation windows. The windows used to estimate the effects of those shocks are shorter than the period covered by the dataset used to estimate the model in (1), which implies that the assumption of a constant effect is not restrictive.

[7] An alternative approach would be to incorporate a shock measure directly into the threshold VAR from the first stage of our analysis (along the lines of proxy VARs). However, such an approach is associated with several methodological questions that are yet to be resolved in the literature. For example, it is not clear how the equivalence between local projections and impulse responses from proxy VARs (Plagborg-Møller and Wolf, 2021) would apply for nonlinear models and for quantile responses instead of standard conditional mean responses.

[8] Nonlinearity is an inherent feature of our modelling framework and would remain so even if we focused on conditional means and examined solely impulse responses within the threshold VAR model. The moving average representation of threshold VARs is nonlinear in the disturbances due to the possibility of a regime change. As a consequence, conditional mean responses can differ depending on the sign and size of the shocks. That is why we conduct an examination of asymmetric and nonlinear specifications.

of these three explanations applies is of extreme relevance to policymakers, who view increased volatility differently from an increase in the asymmetry of the distribution. This is because these three changes imply a different likelihood of "compensation" in the form of above-average output growth, and each therefore prescribes a different policy response. Being able to shed light on these alternative explanations is one of the benefits of our approach that analyzes distribution moments instead of the particular quantiles of the output growth distribution.

Another advantage of our focus on moments rather than quantiles is that it allows us to report the statistical significance of the difference between the conditional mean effect and the tail response. This difference is not usually tested statistically, and the tail effect could in principle be the dominant part of the story. Examining the effect on centered and standardized distribution moments can help us to understand the role of different moments and provide statistically significant evidence of distinct behavior across quantiles.

3. Data and Shocks

The vector of endogenous variables y_t consists of one financial and three macroeconomic variables (all series have been downloaded from the FRED database). The latter refers to real GDP growth (annualized quarter-on-quarter change in real GDP), consumer price inflation (annualized quarter-on-quarter change in the CPI), and the short-term interest rate (the 3-month interbank rate). They capture the key macroeconomic features of a closed economy. The financial sector is represented by the spread between the 10-year bond yield and the Federal Funds rate. It has been included because the slope of the yield curve is often a predictor of future economic activity, see e.g. Estrella and Trubin (2006). In addition, the spread (together with the short-term interest rate) can account for post-2008 unconventional monetary policy measures implemented through large-scale asset purchases, see for example Baumeister and Benati (2013).[9] As a robustness check, an alternative measure of credit conditions is employed as well, specifically the Chicago Fed National Financial Conditions Credit Subindex. Its comparison with the benchmark specification (the 10th percentiles of both the conditional and unconditional output growth distribution) is presented and discussed in Appendix C.3.

When analyzing the conditional moments in Section 0, we employ the economic and policy shocks available in the literature. Monetary policy shocks are taken from Wieland and Yang (2020), who extend the series of narratively identified shocks by Romer and Romer (2004) until 2007Q4. Given that the first estimate of the moments from the model in (1) is available for 1994Q2, the regression in (4) is estimated for the period 1994Q1-2007Q4. In addition, Gertler and Karadi's (2015) monetary policy shocks – available until 2012 Q2 – are employed as a robustness check. Financial shocks are proxied by the exogenous part of the excess bond premium taken from Gilchrist and Zakrajšek (2012), regularly updated to cover the period 1973Q1-2019Q1 (for details see Favara et al., 2016). The regression in (4) is thus estimated for 1994Q1-2019Q1 period.

[9] Note that the spread enters the vector of endogenous variables as a negative of the spread – the reasoning can be found in Appendix A.5.

4. Estimation Procedure of Output Growth Distributions and the Implied Risk Measures

The model in (1) is estimated via the Gibbs sampler. The estimation procedure follows a standard Bayesian estimation of threshold vector autoregressions (Chen and Lee, 1995), extended for the estimation of weights in the threshold variable (Chen and So, 2006). The application of threshold VARs into macro-financial modelling was introduced in Balke (2000). Details of the estimation procedure are provided in Appendices A.1-A.3.

As mentioned above, one of our econometric contributions is a novel sampling approach for the threshold and weights within the Gibbs sampler, i.e. for the parameters whose conditional posteriors are not analytically tractable. The key is to sample from the multinomial distribution instead of employing the Metropolis step. We exploit the fact that the domains of the threshold value, as well as the weights, are bounded and discretize the domain. Appendix A.4 demonstrates a much higher efficiency of the approach relative to the conventional approach. Another addition to this literature is the discussion of the threshold variable including strongly negatively correlated variables – see Appendix A.5.

The model is estimated on windows of 30 years' length. The first estimation window covers the period 1964Q2-1994Q1, and the last estimation window covers the 1989Q2-2019Q1 period. A fixed length of the estimation window is desirable because parameter uncertainty is part of the simulated predictive distributions, so changing the length of the window could spuriously suggest trends in output growth tail risks. For inference, we use 10,000 iterations of the Gibbs sampler that are generated after 5,000 'burn-in' iterations. For the windows with slow convergence of the sampler, a higher amount of iterations is employed and each x-th draw is used for inference. Details can be found in Appendix A.4, together with the convergence diagnostics.

While the primary purpose of the model is to estimate predictive distributions of output growth and the implied tail-risk measures, the estimation results themselves offer several interesting insights into the evolution of US business cycle dynamics. Moreover, they provide an empirical validation of the model in terms of its in-sample properties, and help us to interpret changes in long-run tail-risk over time. The estimation results of the model in (1) can be found in Appendix B.1. Employing a model with two regimes raises the question of how the two regimes differ. Figures B1 and B2 in Appendix B.1 plot the posterior means of the regime-specific steady states and the regime-specific shock volatilities. Figure B2 shows that the estimation windows preceding the GFC are characterized mainly by different shock volatilities across the regimes. In contrast, Figure B1 indicates that the windows containing the GFC and the post-GFC periods are characterized by different steady states, implying the bimodality of the long-run output growth distribution. A more detailed discussion of the regimes and their comparison with the existing literature can be found in Appendix B.1, with an out-of-sample validation of the nonlinear model discussed in Appendix B.4.

During the Gibbs sampler procedure, each drawn vector of the model parameters is used to simulate the model 1,048 periods ahead, yielding an estimate of predictive distributions at various horizons.[10] The simulated predictive distributions are then used to estimate distribution moments and measures of output growth tail risk. The first two moments are represented by the mean and variance of the

[10] The predictive distributions based on the model in (1) are essentially a mixture of two normals with time-varying weights dependent on macroeconomic variables.

simulated values of output growth. The third and fourth moments are represented by the measures of skewness and kurtosis less sensitive to outliers as introduced in Groeneveld and Meeden (1984) and Hogg (1974) respectively.

Regarding the risk measure, we focus on downside tail risk as represented by the 10th percentile of the output growth distribution. As a robustness check, a comparison of the 1st and the 10th percentile of the output growth distribution is reported in Appendix C.4. It turns out that the specific choice of the percentile to summarize the tail of the output growth distribution is not crucial for the results.

As discussed above, an important feature of our analysis is the attention to risk from both the short-run and long-run perspective. The short-term measure of macroeconomic risk is conditional on data in period $t-4$: the 10th percentiles are taken from the predictive output growth distribution four periods ahead. More specifically, we take all simulated paths of output growth and look at the 10th percentile at a distance of four quarters.

The four-quarters-ahead horizon was selected for two reasons. First, the literature examines tail risks at this horizon (see e.g. Adrian et al., 2019). This is because of its relevance for policymakers who possess instruments effective with such (or even longer) delays. Second, the model from the first stage imposes some persistence of the threshold variable, i.e. of the regime the system is currently in. Therefore, the non-normal nature of the simulated distribution can come to life more fully at longer horizons.[11]

Our second measure of macroeconomic risk relates to the long-run output growth distribution. It can be considered unconditional because we utilize all simulated paths of output growth (starting with the 48th quarter to eliminate initial conditions). The length of the simulated path is then 1,000 periods. We take each 20th simulated value to get independent draws from the long-run output growth distribution, and use the 10th percentile from the resulting distribution.

Such unconditional risk measure is not directly related to a specific sequence of shocks occurring during the estimation period of the underlying model. Instead, it encompasses long-term risks related to both the structure of the economy and the policy framework prevailing at the time. In terms of the model, it captures all estimated macroeconomic relationships, shock volatilities and steady states in both regimes. It should be noted that the zero lower bound (ZLB) is imposed during the simulation, i.e. a zero value is forced whenever the simulated interest rate path tends to get into negative territory. In Appendix C.5 we perform a formal check in which the ZLB is not imposed, and show that the selected approach of accounting for the ZLB in the simulation of predictive distributions does not affect our findings.

[11] We also estimated the risk measure for the one-quarter-ahead horizon to check the robustness of our results. While the main results remain unaffected, it turns out that the one-quarter-ahead distributions are closer to the normal distribution than the four-quarters-ahead distributions. Non-normal conditional distributions indeed arise at longer horizons. For details see Appendix B.4.

Both risk measures focus on specific quantiles to streamline the discussion. However, the analysis strongly suggests that if policymakers are to act as risk managers, they need to consider the entire distributions in line with their preferences (loss function).[12]

5. Estimates of US Output Growth Distribution and Risk Measures

In this section, the moments of the estimated conditional and unconditional output growth distributions are reported (Subsection 5.1), and their properties are compared with those in the literature. The implied risk measures are then presented and discussed (Subsection 5.2). The presented descriptive evidence serves as the motivation and basis for the analysis conducted in Sections 6 and 7. Among other things, it suggests the importance of accounting for both the short- and long-run perspectives in modelling macroeconomic tail risk. The differences in the statistical properties of the conditional and unconditional distributions suggest that some policy measures may be beneficial from a short-term perspective, but they may have undesirable longer-term consequences that need to be taken into consideration.

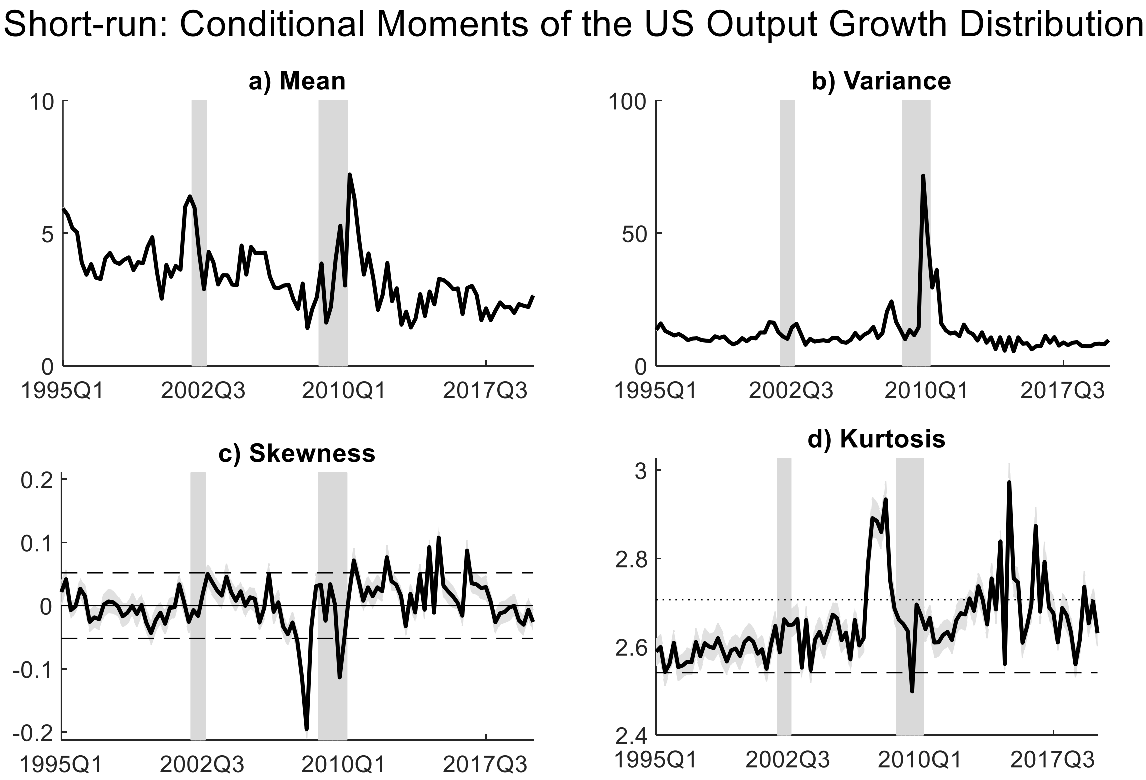
5.1 Output Growth Distributions

Figures 1 and 2 present the first four moments of the conditional and unconditional output growth distributions respectively. Both figures also report the skewness and kurtosis of several standard distributions (dashed and dotted lines) to provide some guidance on the extent of the asymmetry and fat tails.

In order to maintain our focus, we will not discuss all the estimation results in this paper. We will only highlight those that offer some policy-relevant insights or shed new light on existing research findings. In the short-run perspective of Figure 1 the key messages relate to the third moment. Panel c) demonstrates that the conditional output growth distribution is generally asymmetric, and it does not feature fat tails. In particular, it shows that the conditional output growth distribution is negatively skewed before and during recessions. The important lesson for policymakers relates to negative skewness. The analysis implies that negative ex-post output growth surprises (with respect to the estimated conditional mean) are, on average, larger than positive surprises. This means that unless the entire distribution of key macroeconomic variables is taken into account (including the fact that downside risks are larger than upside risks before/during economic downturns), the policy actions may be erroneous. In an attempt to alleviate the existing deficiency in the policy treatment, Section 6 provides a detailed analysis of this asymmetry based on the distributional effects of the structural shock.

[12] For example, Alessi et al. (2014) describe the risk-management approach to monetary policy at the Federal Reserve Bank of New York. Kilian and Manganelli (2008) show what a central banker’s loss function can look like, and how it can be linked to macroeconomic risk represented by (subsets of) distributions of macroeconomic variables.

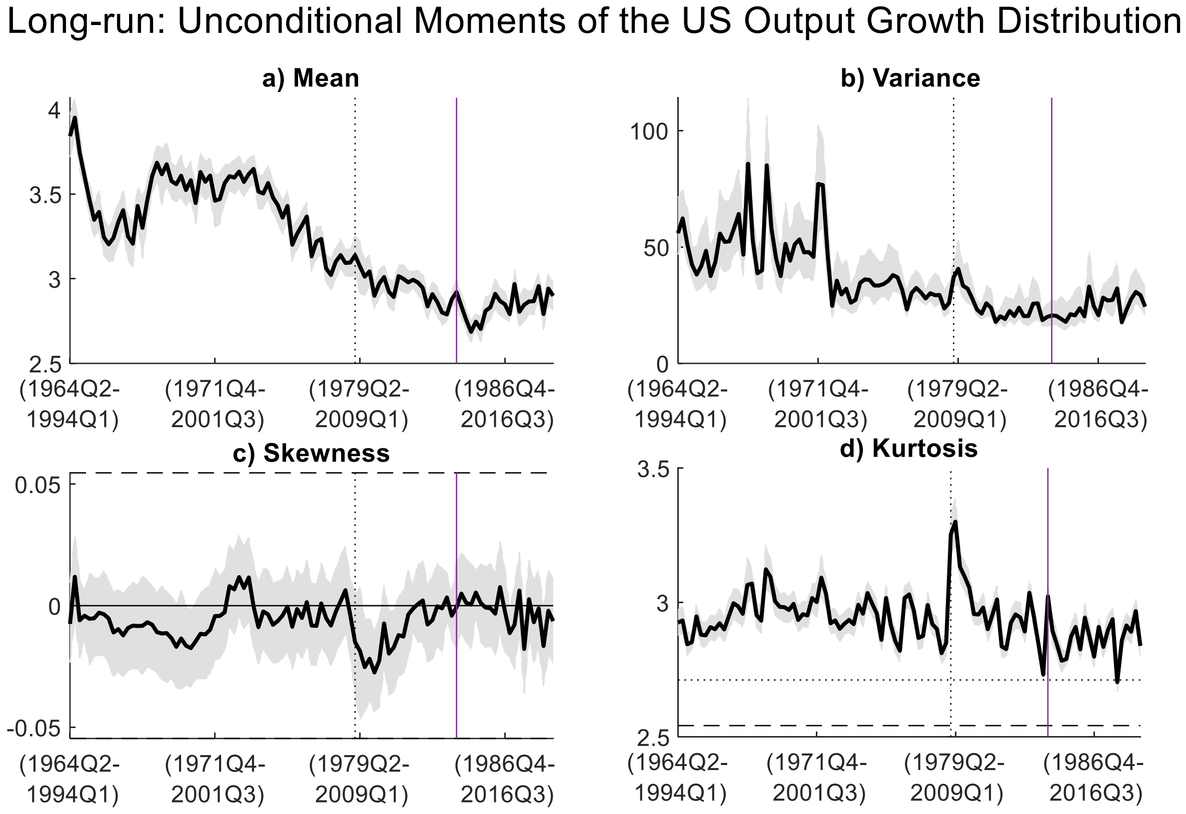
Figure 1: The Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution



Note: The x-axis indicates the quarter of the estimated conditional output growth distribution. The distribution is conditional on the data available four quarters earlier. The shaded areas denote the 90% confidence intervals obtained by bootstrapping from 500 bootstrap samples. The dashed line in panel c) indicates skewness of the simulated lognormal distribution (the negative of) with parameters $\boldsymbol{\mu} = \mathbf{0}, \boldsymbol{\sigma} = \mathbf{0.1}$. The dashed line in panel d) indicates the kurtosis of the simulated standard normal distribution. The dotted line in panel d) indicates the kurtosis of the simulated Student's t-distribution with 10 degrees of freedom. The number of simulated values from the respective distributions equals the number of simulated values of the output growth distribution reported in the respective panel. The vertical shaded areas in all panels represent official NBER recessions.

Moving from the short-run to the long-run perspective, Figure 2 shows how the unconditional moments have evolved over time. Panels c) and d) suggest that the unconditional distribution is closer to being symmetric and it is heavy-tailed, which contrasts the conditional distribution in Figure 1 that was strongly asymmetric with no fat tails. Specifically, in our long-run analysis the statistical significance of a negative skew is found at the time the GFC enters the estimation windows and at the end of the series of estimation windows only. From the policymaker's perspective, the observed asymmetry and fat tails send the message that the distributional consequences of the entire policy framework should be assessed in terms of their long-run effect on risk. We analyze the role of the monetary policy framework on the unconditional output growth distribution moments in Section 7.

Figure 2: The Mean, Variance, Skewness and Kurtosis of the Long-run Output Growth Distribution



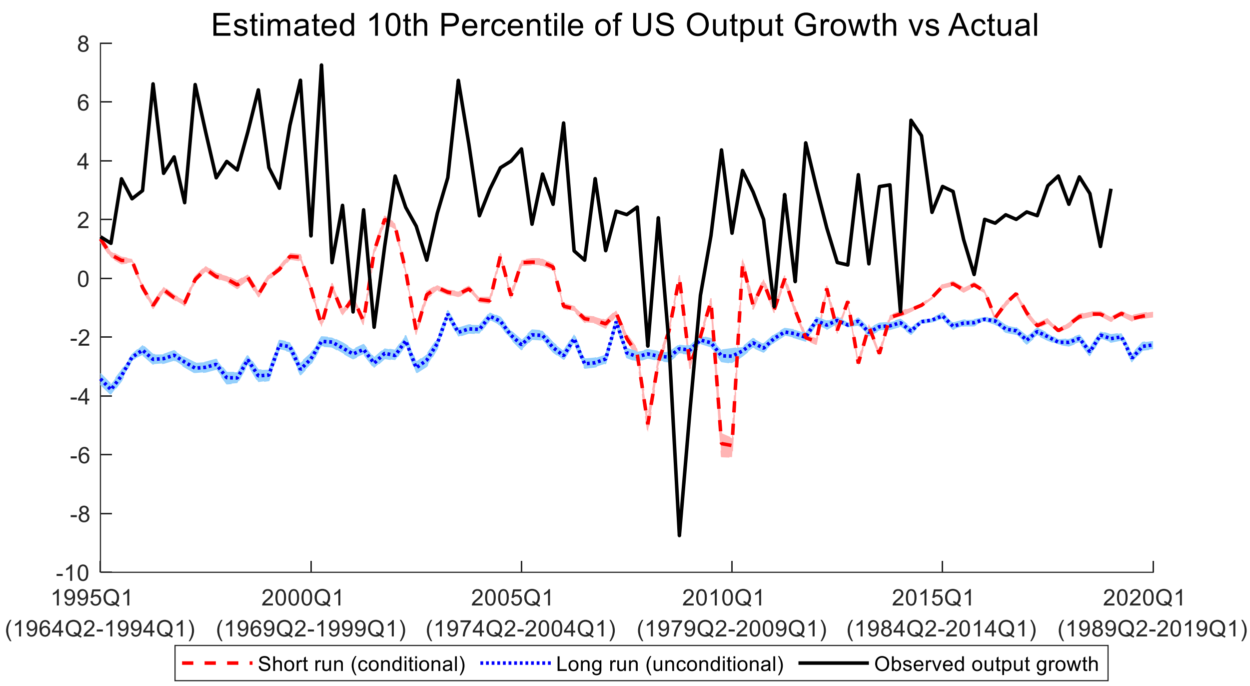
Note: The x-axis indicates the period used to estimate the model employed for simulation of the long-run output growth distribution. The shaded areas denote 90% confidence intervals obtained by bootstrapping from 500 bootstrap samples. The dashed line in panel c) indicates skewness of the simulated the lognormal distribution (the negative of) with parameters $\mu=0$, $\sigma=0.1$. The dashed line in panel d) indicates the kurtosis of the simulated standard normal distribution. The dotted line in panel d) indicates the kurtosis of the Student's t distribution with 10 degrees of freedom. The dotted vertical line indicates the estimation window when the GFC enters the estimation (1979Q1-2008Q4). The vertical line indicates the estimation window 1984Q2-2014Q1, which first includes the post-GFC normalization of monetary policy.

Our ability to reproduce important findings in the literature serves as further justification for our modelling approach. Most importantly, in terms of the third moment, the reported negative conditional skewness before and during recessions is consistent with the studies that employ different approaches (both parametric and nonparametric), for example, Delle Monache et al. (2020), Adrian et al. (2019), and Morley and Piger (2012). The lack of asymmetry of the unconditional distribution is in line with e.g. Carriero et al. (2020). In terms of the fourth moment, the fat tails of the unconditional output growth distribution have been detected by e.g. Fagiolo et al. (2008). We can also replicate two well-known long-term phenomena of the past several decades related to the first two moments. They are a gradual decrease in potential output growth over the past several decades (see e.g. Antolin-Diaz et al., 2017), and a decrease in the shock volatility of macroeconomic variables (known as the Great Moderation, see e.g. Stock and Watson, 2003).

5.2 Risk Measures

Having obtained the estimates of the output growth distribution, we can focus on their 10th percentile that serves as a common left-tail risk measure. Naturally, whenever the 10th percentile falls downside risk rises, and vice versa. Figure presents two variants of the estimated percentile, representing downside risk in the short run (the red dashed line) and in the long run (the blue dotted line). As a reference point, ex-post observed output growth is also plotted in Figure 3 (the black solid line).

Figure 3: Estimates of the 10th Percentile of the Short-run (Four-quarters-ahead) and the Long-run US Output Growth Distribution, together with Ex-post Observed Output Growth



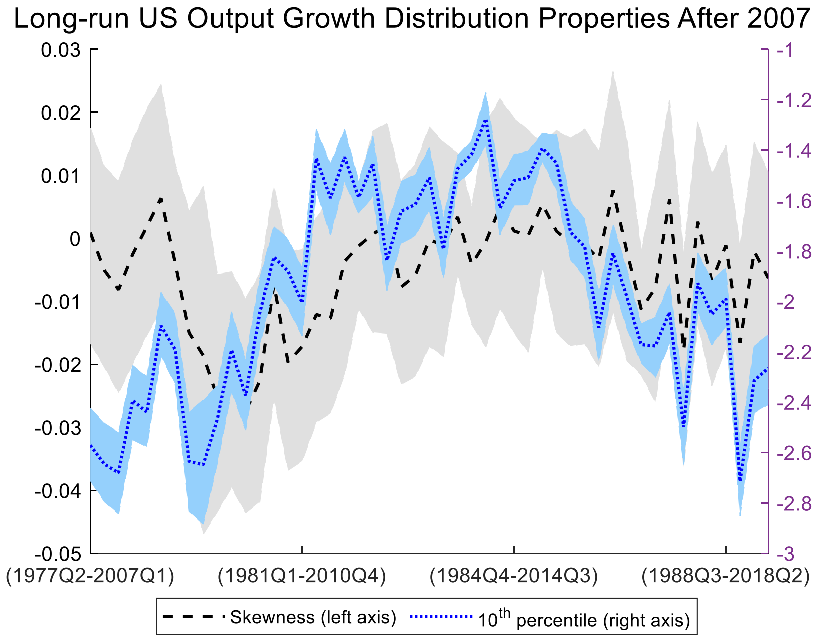
Note: Real output growth is in the form of annualized quarter-on-quarter percentage changes. The x-axis indicates the quarter for which the 10th percentile of the short-run output growth distribution is estimated and ex-post output growth observed. The periods in parenthesis indicate the windows used to estimate the long-run output growth percentile. The shaded areas indicate the 90% confidence intervals generated from 500 bootstrap samples (they are very narrow and thus barely visible).

In terms of the short-run perspective, there are no obvious trends: the 10th percentile of the conditional output growth distribution follows the business cycle. The fall of the 10th percentile prior to and during the GFC reflects the above-discussed change in conditional skewness. It implies that the rise in short-run risk was greater than what could be explained by changes in expected output growth (the conditional mean) alone.

Long-run risk has however followed a different trajectory that exhibits trends. The initial two thirds of the estimation windows until the GFC can be broadly characterized by an increasing trend in the 10th percentile of the unconditional output growth distribution, i.e. the long-run downside tail risk was declining. This trend reversed around 2014, after which long-run risk has been steadily increasing. This is more clearly visible in Figure 4 below, which plots a subset of the estimates since the GFC (together with skewness).

Figure 4 makes apparent that the increase in the tail risk after 2014 is substantial. The 10th percentile decreases from values of around -1.3% to close to -2.7% (the right axis). It means that 10% of the time recording the worst economic performance, the decline in output growth will be approximately 1.4 percentage points greater.

Figure 4: The Skewness and the 10th Percentile of the Long-run US Output Growth Distribution



Note: The x-axis indicates the period used to estimate the model employed for simulation of the long-run output growth distribution. The shaded areas denote 90% confidence intervals obtained by bootstrapping from 500 bootstrap samples.

Figure 4 also reveals the important role of skewness for the post-GFC developments of long-run downside tail risk; relative to other moments reported in Figure 2. In 2009-2014 (the interval between the dotted and solid vertical lines in Figure 2) the decline in long-run risk is accounted for by the changes in skewness, as well as a fall in the mean of the output growth distribution. The reversal and subsequent rise in the long-run risk measure post-2014 is due to a combination of an increase in the variance and a decrease in the skewness. Both of these adverse changes push the left tail downwards, and they thus outweigh the positive influence of an increasing unconditional mean on the left tail of the output growth distribution.

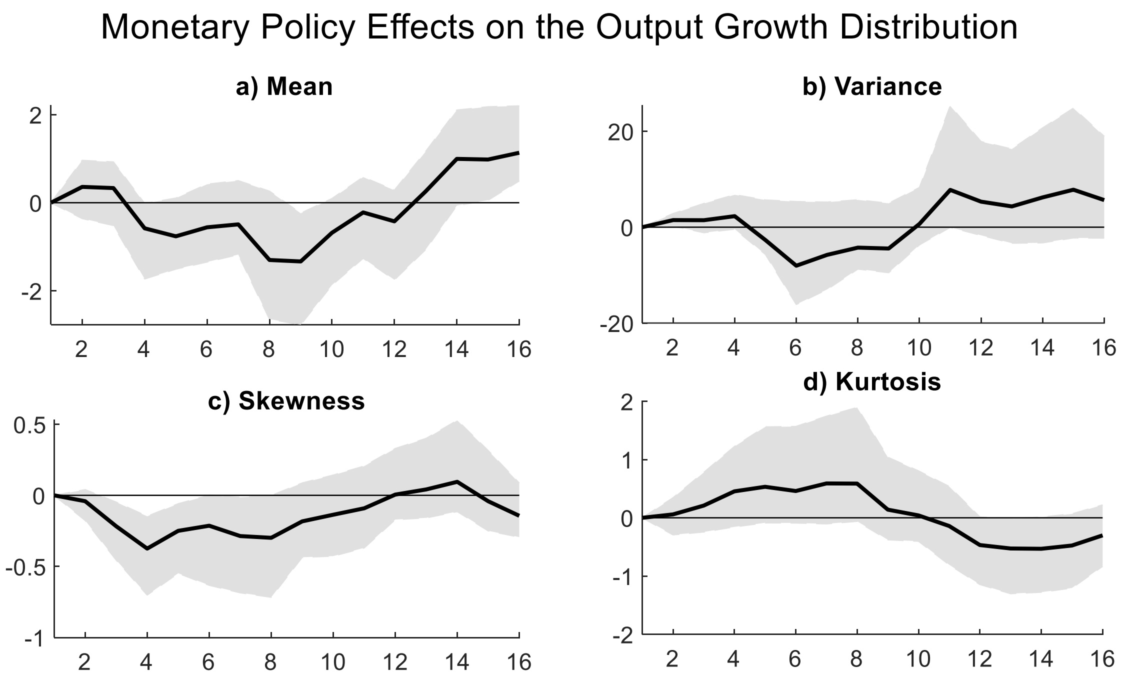
6. Analysis of Short-run Risk

To shed some light on short-run macroeconomic tail risk, we examine the effects of two adverse shocks on the conditional output growth distribution moments. Subsections 6.1 and 6.2 investigate monetary policy shocks and financial shocks respectively, and both compare the estimated effects with the existing literature. The analysis finds that while the effect of shocks on the conditional mean is comparable to what has been reported in the literature, the tails behave differently from the mean because the conditional distribution becomes asymmetric. Moreover, the two shocks affect the symmetry of the conditional distribution differently. In contrast, the impact of both considered shocks on fat-tailedness is negligible.

6.1 Monetary Policy Shocks

Given the importance of the monetary policy shock for the dynamics of US macroeconomic variables (see e.g. Liu et al., 2019), we make it our starting point. Figure 5 shows the effect of a 100 basis points (b.p.) monetary policy tightening on the conditional mean, variance, skewness and kurtosis of the four-quarters-ahead output growth distribution.

Figure 5: Response of the Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution to an Unexpected 100-basis-points Monetary Policy Tightening



Note: The shaded areas indicate the 90% confidence intervals.

In regards to the conditional mean, the effect of an unexpected monetary policy tightening, as depicted in panel a) of Figure 5, is consistent with the literature in terms of its direction – there is a fall in output growth. However, in contrast to the literature, the effect is longer-lived. A statistically significant decrease of the conditional mean of four-quarters-ahead output growth is estimated at a horizon of over 2 years, whereas most of the literature suggests a maximum effect of 1-2 years (see e.g. Christiano et al., 1999). Our finding is however in line with studies implying that such longer-lived effect of monetary policy may indeed occur due to various hysteresis mechanisms (see e.g. Jordà et al., 2020).[13]

[13] From a modeling point of view, the persistent effect is a consequence of the mean adjustment employed in the model in the first stage. The mean adjustment drives behavior over the longer term and thus makes the model more realistic for policy analysis.

Delving deeper, our estimation results indicate that conditional skewness plays a vital role in short-run risk. Skewness decreases within a two-year horizon and then increases again. This means that short-run left-tail risk (the probability of very low output growth) temporarily increases and then returns to its pre-shock level. The change in the asymmetry of the distribution is statistically significant implying that the left tail temporarily diverts from the conditional mean (and from the right tail too). The short-run resilience of the economy thus decreases by more than what would be implied by the conditional mean alone. This is a message to risk-averse monetary policymakers whose set of objectives include output growth stabilization. Broadening the focus from the mean to the entire output growth distribution implies that (ceteris paribus) monetary policy, both its tightening and easing, should be less aggressive.[14]

Regarding the even moments (variance and kurtosis), the results show that monetary policy shocks have only minor effects that can be considered robust across our specifications and shock measures (for some nonlinear specifications see Figure C1-C3, Appendix C.1 also includes a discussion on the exogeneity of shocks). One can therefore conclude that monetary policy shocks do not symmetrically affect the probability of (extremely) high and low output growth in the short term.

The literature usually finds that monetary policy tightening has a more profound effect on the left tail of the output growth distribution represented by the 5th or 10th percentiles in comparison with its effect on the conditional mean/median (Duprey and Ueberfeldt, 2020, Loria et al., 2019, Jung and Lee, 2019). Our analysis of the moment responses reveals that the temporary increase in output growth tail risk after a monetary policy tightening is due to the conditional mean effect, as well as due to an increased asymmetry of the output growth distribution. The contribution of both factors (odd moments) is statistically significant. In contrast, conditional variance and kurtosis (even moments) play a secondary role.

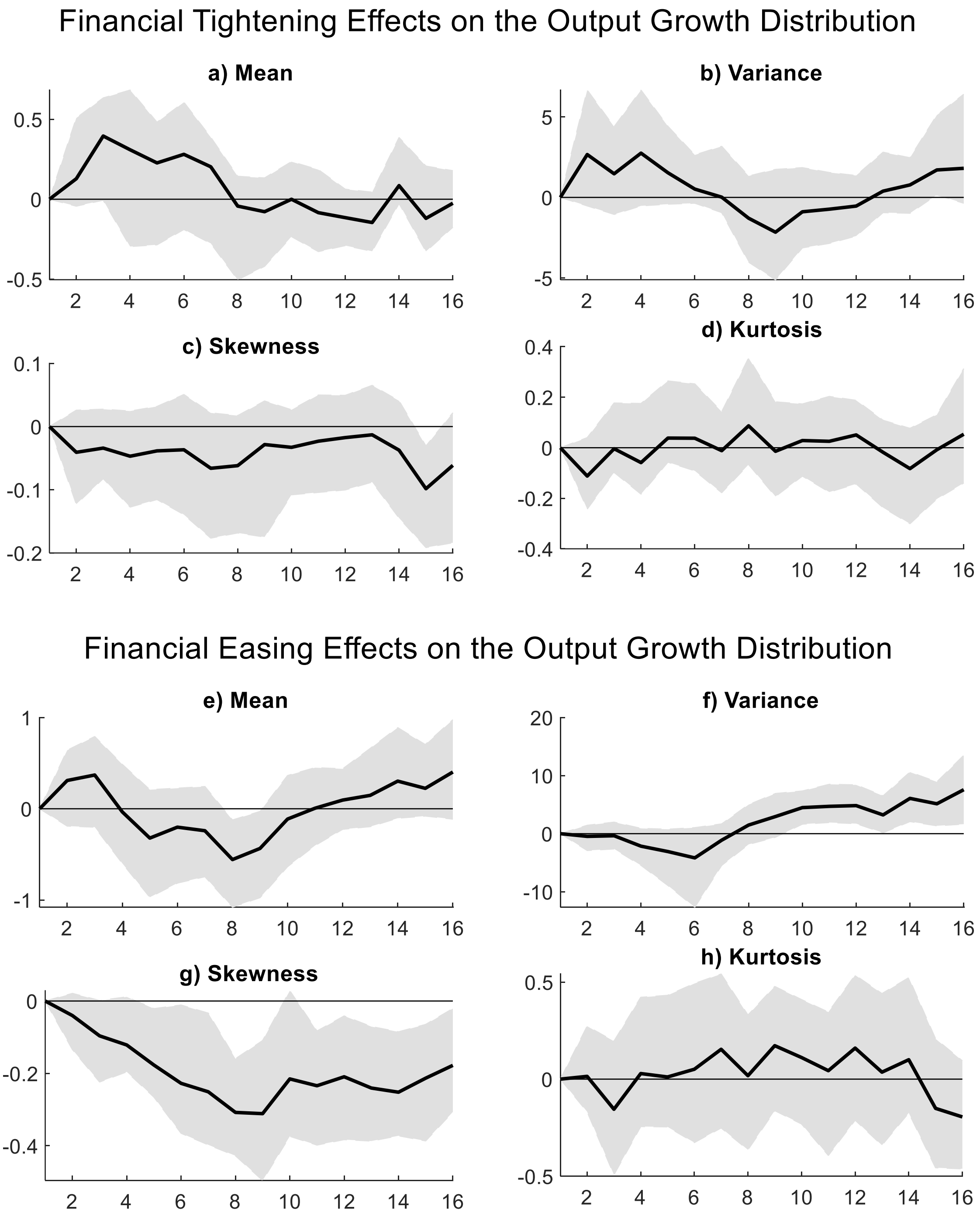
6.2 Financial Shocks

The second type of shock employed in our analysis is a financial shock represented by the credit spread, i.e. unexpected changes in the excess bond premium. In contrast to the narratively identified monetary policy shocks described above, the excess bond premium changes cannot be viewed as a source of exogenous variation. We follow Loria et al. (2019) and extend the regression equation in (4) to the contemporaneous Federal Funds rate, contemporaneous output growth, contemporaneous inflation and four lags of the excess bond premium in order to control for the premium's endogenous movement.

Contrary to the monetary policy shocks, the effects of financial shocks on the output growth distribution moments are not symmetric. To illustrate this, Figure 6 presents separately the effects of a tightening of financial conditions (an increase in the risk premium) and of financial easing (a decrease in the risk premium). The asymmetric effects represent a stylized fact brought about by strong nonlinearities between the financial conditions and the real economy (Akinci and Queralto, 2017). Note that in general the financial shocks we examine can either be attributed to events in the financial sector or to macroprudential policy itself (through, for example, imposing bank capital requirements or risk weights).

[14] A simple general equilibrium framework with this feature can be found in e.g. Duprey and Ueberfeldt (2020).

Figure 6: ***Response of the Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution to an Unexpected 100-basis-points Change in the Excess Bond Premium***



Note: Panels a) to d) show a tightening of financial conditions; panels e) to h) show their easing. The shaded areas indicate the 90% confidence intervals. The separate policy shocks are employed in a way that if only tightening shocks are considered, unexpected easing is represented by 0, and vice versa.

Figure 6 shows an increase in the conditional variance primarily after an unexpected easing of financial conditions, implying an increase in short-run tail risk. A similar increase in output growth variance is also observed in the quadratic specification, which confirms that both financial tightening and easing affect output growth variance in the same direction (see Figure C5 in Appendix C.2). This second moment effect is further strengthened by the shift of conditional skewness towards a more negative skew, which is substantial under financial easing.

Comparing the effects of monetary policy and financial shocks suggests an important difference between conventional and unconventional monetary policy measures. The latter measures, based on quantitative or credit easing, are shown to weaken the resilience of the economy in the short-term – the conditional skewness decreases. The opposite is true when easing is delivered by conventional monetary policy actions.

Similarly, macroprudential measures that work through credit spreads could in principle adversely affect the left-tail risk of output growth over and above the conditional mean. We demonstrate that such measures could do so through three avenues: a decrease in the conditional mean, an increase in conditional variance, and through more negative conditional skewness. It should be mentioned that the transmission via credit spreads only captures one of the potential channels of macroprudential policy, so the overall effect cannot be unambiguously established. Despite this, our results imply a warning about the possible adverse consequences of macroprudential easing for downside output growth risk.

Our estimates go hand in hand with the findings reported in the literature, but they provide a novel context and/or an underlying explanation for them. Adrian et al. (2019) found that deterioration of financial conditions represented by the National Financial Conditions Index relates to a fall in the conditional mean and an increase in the conditional volatility. This is consistent with a fall in the lower quantiles while the upper quantiles remain stable. Our analysis enriches this message by showing that a fall in the lower quantiles is not only due to the conditional mean and volatility, but also due to skewness, which makes the distribution asymmetric with a longer left-tail. Furthermore, we show that it is the easing of financial conditions that has a larger effect on the conditional distribution than the tightening.

Another substantiation of existing research findings relates to Aikman et al. (2019). The authors find that credit booms increase downside risk over the 3-5 year horizon. Looking at our estimated impact of loosening financial conditions in Figure 6 and assuming a link between a fall in the credit spread and a credit boom, our analysis implies that the increase in downside risk is due to two factors. One is an increase in volatility while the other is a greater negative skewness. It thus follows that the fall in Growth-at-Risk is not fully compensated on the upside. This provides a novel empirical argument for risk-averse policymakers to impose more stringent financial market regulatory measures during exuberant credit booms.

Finally, Loria et al. (2019) find that one year after a credit spread shock the conditional distribution narrows, i.e. the 10^{th} and 90^{th} quantiles get closer to each other. Our analysis offers an explanation for this finding. It is likely to be a consequence of the fall in conditional volatility when the tightening and easing effects are aggregated within a linear specification. In such cases the effect of easing on volatility dominates, and as easing enters the linear specification with a minus sign a fall in volatility is observed. Our analysis hence implies that distinguishing between credit easing and credit tightening is essential for devising appropriate risk-management policies.

7. Analysis of Long-run Risk

In this section, we examine the profile of the 10^{th} percentile of the unconditional output growth distribution and the implied long-run downside tail risk. In contrast to the above short-term perspective, which related to shocks and specific policy actions within a given policy framework, the long-run insights relate to the choice of policy framework as such. We provide two arguments that link the post-GFC developments in long-run downside tail risk to the US monetary policy regime.

The first argument draws strictly on the model in (1) and should thus be interpreted as being of a reduced-form nature. It is based on estimated (reduced-form) policy rules, so it relates to the systematic component of the observed policies. The second argument draws on our short-term analysis in Section 6. In this respect, it is of a structural nature, employing information from outside the model in (1) in the form of structural shocks provided in the literature. As a consequence, the latter approach is more informative about the nonsystematic (unexpected) component of the policies.

We show that both the systematic and nonsystematic components of monetary policy prior to and after the GFC help to explain the change in the long-run tail risk. The takeaway for policymakers is that the choice of policy framework matters for long-run downside macroeconomic tail risk. Frameworks that imply an asymmetric distribution should be complemented by policies that can address and rectify the resulting problems. In the post-2008 era, targeted macroprudential regulation that responds to a negatively skewed output growth distribution brought about by unconventional monetary policy (quantitative easing) is a case in point. Our analysis implies that such macroprudential regulation may be essential to prevent subpar output growth further down the track. It could take the form of higher capital requirements and buffers for example.

7.1 Long-run Risk and Systematic Policy

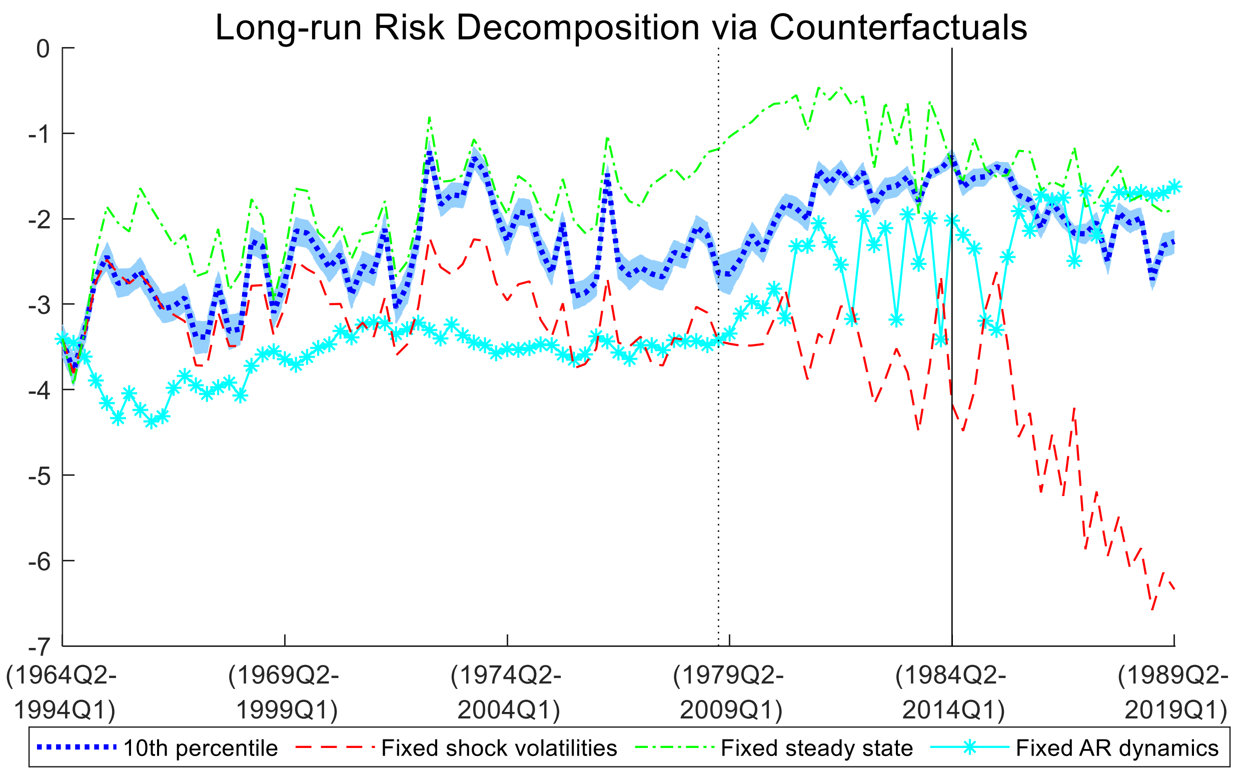
The first step in analyzing the 10^{th} percentile of the long-run output growth distribution is to construct counterfactuals that help uncover the role of macro dynamics, shock volatility and steady states respectively. We focus on the 10^{th} percentile rather than the distribution moments because the exposition is more straightforward, but a similar exercise is conducted for the moments of the output growth distribution and reported in Appendix B.2.

The counterfactuals are constructed by keeping constant various subsets of the whole parameter vector in model (1), one at a time.[15] Figure 7 plots the three counterfactuals we examine: fixed reduced-form shock volatilities, fixed steady states and fixed autoregressive parameters. For example, the first counterfactual shows what the 10^{th} percentile would be if the steady states in both regimes were kept unchanged over time. The second and third counterfactual conduct the same exercise, fixing the regime-specific shock volatilities and the regime-specific short-run dynamics

[15] The counterfactuals draw on a reduced-form model, and hence the regime-specific error covariance matrices include the contemporaneous effects of structural shocks on endogenous variables. As such, imposing reduced-form shock volatilities imposes part of the short-run dynamics. In what follows, short-run dynamics thus means reduced-form AR parameters, and shock volatilities mean reduced-form shock volatilities.

respectively. Comparing the counterfactual with the factual then indicates whether the specific feature (subset of parameters) contributed to the observed profile of tail risk, and in what way.[16]

Figure 7: The 10th Percentile of the Long-run Output Growth Distribution and the Counterfactuals with a Fixed Steady State, Fixed Shock Volatilities and Fixed AR Parameters Respectively



Note: The x-axis indicates the period used to estimate the percentile in the long run. The shaded area indicates the 90% confidence intervals. The dotted vertical line indicates the estimation window in which the GFC enters the estimation (1979Q1-2008Q4). The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

Let us first focus on the pre-GFC period (the first half of the estimation windows) in Figure 7. The increasing trend in the 10th percentile, i.e. the decline in downside risk, is mainly due to the short-run dynamics (the blue line with stars) and partly also due to shock volatilities (the red dashed line). Intuitively, keeping both unchanged would lead to a lower 10th percentile, i.e. higher downside risk. The role of shock volatilities becomes more profound in the middle of the series of the estimation windows, and it increases over time. Finally, the steady states (the green dash-dotted line) drives the 10th percentile towards lower values (higher risk), but the overall effect is more than offset by the other two factors.

The most policy-relevant finding in Figure 7 however relates to the post-GFC evolution in long-run macroeconomic tail risk (the second half of the estimation windows). A slight decline in downside

[16] More specifically, given an estimation window we use the respective output from the Gibbs sampler, i.e. draws of all subsets of model parameters except the one that is fixed over all estimation windows (and taken from the first estimation window). Then we simulate the model again 1,048 periods ahead to obtain the long-run risk measure as in the benchmark case. This is conducted for all estimation windows.

risk can be observed when the GFC enters the estimation windows, but this is followed by an increase in risk when post-2014Q1 data enter the estimation windows, i.e. after the Fed started normalizing its monetary policy. The counterfactual with fixed AR parameters (the blue line with stars) exhibits an increase in the 10^{th} percentile, which demonstrates the prominent role played by short-run dynamics in the rise in long-run downside tail risk after 2014.

To shed some light on the nature of the change in the AR dynamics after 2008, Figures B4 and B5 in Appendix B.1 present the evolution of the AR coefficients over the estimation windows in the interest rate equation and in the spread equation of the model in (1). Remarkable qualitative changes in the estimated coefficients occur with the recent changes in long-run tail risk apparent in the estimation windows 1979Q1-2008Q4 (start of the GFC) and 1984Q2-2014Q1 (start of the monetary policy normalization). The change is abrupt especially for the AR coefficients related to output growth; see panels a) in Figures B4 and B5. In Appendix B.1, we discuss in detail the fact that this can be interpreted as a change in the monetary policy regime. When the GFC enters the estimation windows (post-2008Q4), the regime can be characterized by the monetary authority placing a low weight on both output growth and inflation within its interest rate policy. In contrast, the post-2014Q1 period can be characterized as a policy regime in which the monetary authority assigns a low weight on inflation and a high weight on output growth. The relationship between the spread and lagged output growth also undergoes a significant qualitative change. It turns from positive to negative, whereby the switch coincides with the 1984Q2-2014Q1 estimation window.

The characteristics of the regimes correspond to Bianchi (2013). His estimates indicate that the 1960s, the 1970s and the GFC can be characterized as the "Dove" regime, involving an interest rate policy with a low weight on inflation, whereas the rest of the periods can be described as the "Hawk" regime with a high weight on inflation. We expand on this by showing that the post-2014 period features the "Dove" regime with an increasing weight on output.

The changes in the parameters of model (1) suggest that the post-GFC period represents a major departure from earlier policy frameworks. Inflation has become less important for policymakers, and instead monetary policies have been linked more strongly to the performance of the real economy and financial conditions. The counterfactual with fixed AR parameters shows that the increase in the unconditional tail risk towards the end of the sample is associated with the change in the short-term dynamics of macroeconomic variables. Such change can be described as a monetary policy regime change, and it represented a source of elevated downside tail risk in the long run.

7.2 Long-run Risk and Nonsystematic Policy

In Section 6, we discussed the effects of monetary policy and financial shocks on the moments of the conditional output growth distribution. In terms of the monetary policy shock (an unexpected interest rate change), policy easing and tightening had the same distributional effects (with an opposite sign). The credit spread shock was however different in nature. Both credit spread easing and tightening resulted in a negatively skewed distribution and led to a higher variance.

It follows that policy frameworks based predominantly on the interest rate as an instrument tend to imply a symmetric unconditional output growth distribution, whereas those based on the credit spread lead to an asymmetric and more volatile distribution. The rolling windows analysis described in detail in Appendix B.5 reveals that the average size of the credit spread shock is negatively correlated with the skewness of the unconditional output growth distribution (the correlation coefficient is equal to -0.42 and is statistically significant at the 1% level). This suggests that the policy framework based on credit spread shocks is associated with a skewed unconditional distribution. No such statistical relationship is detected for monetary policy (interest rate) shocks, suggesting that conventional monetary policy does not affect unconditional output growth symmetry.

These findings are highly relevant for the post-2008 period. In the aftermath of the GFC the monetary policy framework has been largely based on the credit spread. Quantitative easing influences credit spreads through the portfolio rebalancing channel, whereby its direct impact on yield spreads spills over to close substitutes of safe assets bought and sold by the Fed. Our findings thus imply that the post-GFC policy has made the long-run output growth distribution asymmetric and increased its variance.[17]

The established link between the policy framework and the long-run output growth asymmetry sheds light on two long-run phenomena. First, it explains why the unconditional output growth distribution is symmetric before the GFC enters the estimation windows. Second, it explains why the unconditional output growth distribution has become asymmetric (with a higher variance) after the GFC enters the estimation windows (see Figure 2, panel c). As we discussed in Section 5, the more negative skewness and the increase in variance explain the rise in downside tail risk since 2014, which provides the link between credit spread shocks and policy-induced tail risk.

8. Summary and Conclusions

Almost two and a half thousand years ago the Greek philosopher Herodotus observed that “great deeds are usually wrought at great risks”. Researchers and policymakers have attempted to better understand these risks to assist people and entire economies in achieving their potential, i.e. in accomplishing great deeds. This paper is another such attempt. It proposes an econometric approach to estimating and analyzing tail risks, which can provide valuable insights into past macroeconomic developments as well as into effective policy measures.

Our exploration is divided into two stages. In the first stage, we put forward a new empirical framework to estimate the evolution of the entire US output growth distribution. The framework (which can be applied to other economies and variables as well) allows for asymmetry and fat tails of the distribution, i.e. it relaxes the conventional Gaussian assumption. Importantly, the framework features a mechanism that disciplines its long-run behavior. Therefore, the long-run output growth distribution and risk can be analyzed over and above the standard short-run tail risk measures built on conditional quantiles of the output growth distribution. Adding the long-run perspective – in a way that is consistent with the short-run perspective – is one of the contributions of this paper.

The second stage of the analysis consists of an empirical examination of the derived US output growth distributions. We exploit the fact that the entire distributions are simulated to explore the

[17] There is evidence supporting our argument that the nonsystematic component of monetary policy based on the interest rate implies the dominance of interest rate shocks hitting the economy, whereas monetary policy based on the credit spread results in the dominance of credit spread shocks. Most notably, focusing on the US, Liu et al. (2019) found the dominance of interest rate shocks during the late 1990s and mid-2000s, as well as the importance of the effects of yield spread shocks after the GFC.

effect of monetary policy and financial shocks on the moments of the conditional output growth distribution. This extends the literature that has generally only considered a handful of selected quantiles of the distribution. Being able to distinguish which distribution moment is behind the effect of a certain shock on various quantiles of the output growth distribution (most importantly the tails) can help policymakers to tailor their actions more effectively.

We then take advantage of our nonlinear model from the first stage to interpret the changes in the simulated distributions from a long-run perspective. It is not conditional on any short-term shocks and thus encapsulates the ongoing trends and structural developments. In particular, we examine the time profile of the long-run output growth distribution, and long-run output growth tail risk, by decomposing the distinct effects of changes in the steady states of macroeconomic variables, changes in their shock volatilities, and changes in short-run macroeconomic dynamics. Such decomposition can shed some light on the general role of various economic frameworks and structural factors for output growth tail risk.

The main broad message is that the entire distribution of key macroeconomic variables should be taken into account, because various policy and economic/financial shocks affect the conditional mean differently from the tails. Furthermore, both the short- and long-run perspectives on downside macroeconomic risk need to be examined, as both offer valuable (and sometimes conflicting) information. Most relevantly, our analysis shows that the GFC and the 2014 normalization of US monetary policy represent important regime changes that can be linked to a rise in long-run output growth tail risk in the past several years.

A number of empirical checks have been conducted to ensure that our results are robust. They imply that the methodological innovations and macroeconomic insights in this paper could potentially help to improve real-world policymaking and society's wellbeing.

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Appendix A: Bayesian Estimation of the TVAR Model

This appendix discusses the estimation procedure of the model introduced in Section 2.1. We present the model in the vector form (Section A.1), describe the likelihood and priors (Section A.2), and examine the resulting conditional posteriors and the Gibbs sampler (Section A.3). Finally, the convergence of the sampler and issues related to its efficiency are discussed (Sections A.4 and A.5).

A.1 Model

The model in (1) can be reformulated into a vector form as follows:

$$\bar{y}^{(r)} = \bar{X}^{(r)}\beta^{(r)} + \bar{Z}^{(r)}\delta^{(r)} + \varepsilon^{(r)} \qquad \text{for } r = 1, 2, \tag{A1}$$

where $\varepsilon \sim N\left(0, \bar{\Sigma}^{(r)}\right)$. Defining $Y^{(r)}$ as a $t^{(r)} \times n$ matrix of observations related to regime *r*, then the $nt^{(r)} \times 1$ vector $\bar{y}^{(r)}$ is defined as $\bar{y}^{(r)} \equiv vec\left(Y^{(r)}\right)$. Next, defining $X^{(r)}$ as a $t^{(r)} \times np$ regime-specific matrix that comprises up to p lags of the vector of endogenous variables, the $nt^{(r)} \times n^2p$ matrix $\bar{X}^{(r)}$ is defined by the following Kronecker product: $\bar{X}^{(r)} \equiv I_n \otimes X^{(r)}$. The $n^2p \times 1$ vector $\beta^{(r)}$ is defined as follows:

$$\beta^{(r)} = vec\begin{pmatrix} \left(A_1^{(r)}\right)' \\ \vdots \\ \left(A_p^{(r)}\right)' \end{pmatrix}. \tag{A2}$$

The $nt^{(r)} \times n(p+1)$ matrix $\bar{Z}^{(r)}$ equals to $I_n \otimes Z^{(r)}$, where $Z^{(r)}$ is a $t^{(r)} \times (p+1)$ matrix of the following constant terms:

$$Z^{(r)} = \begin{bmatrix} 1 & -1 & \cdots & -1 \\ 1 & -1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & -1 & \cdots & -1 \end{bmatrix}. \tag{A3}$$

Finally,

$$\delta^{(r)} = vec\begin{pmatrix} F^{(r)\prime} \\ F^{(r)\prime}\left(A_1^{(r)}\right)' \\ \vdots \\ F^{(r)\prime}\left(A_p^{(r)}\right)' \end{pmatrix} \tag{A4}$$

is an $n(p+1) \times 1$ vector including the steady states of the endogenous variables. The vector of the model parameters is then the following:

$$\boldsymbol{\theta} \equiv \left[\beta^{(1)}, \beta^{(2)}, F^{(1)}, F^{(2)}, R, w, \bar{\Sigma}^{(1)}, \bar{\Sigma}^{(2)}\right].$$

A.2 Likelihood and Priors

The estimation procedure draws on the Bayesian approach, i.e. a posterior distribution is formulated based on the likelihood of data and a joint prior distribution. The likelihood function of the model in (A1) is:

$$\pi(\bar{y}|\theta) = \left|\bar{\Sigma}^{(1)}\right|^{-\frac{1}{2}}\left|\bar{\Sigma}^{(2)}\right|^{-\frac{1}{2}} *$$
$$* \, exp\left\{-\frac{1}{2}\sum_{r=1}^{2}\left(\bar{y}^{(r)} - \bar{X}^{(r)}\beta^{(r)} - \bar{Z}^{(r)}\delta^{(r)}\right)\left(\bar{\Sigma}^{(r)}\right)^{-1}\left(\bar{y}^{(r)} - \bar{X}^{(r)}\beta^{(r)} - \bar{Z}^{(r)}\delta^{(r)}\right)\right\}. \quad \text{(A5)}$$

The prior distribution is set as follows: the vectors of the autoregressive coefficients $\beta^{(1)}$ and $\beta^{(2)}$ are distributed normally around zero and 0.9 respectively for the first lag of the interest rate in the interest rate equation. The interest rate enters the vector of endogenous variables in levels. The variance of the prior distribution is of the Minnesota-type prior, that is, a diagonal matrix. The element corresponding to the *i*-th equation and (each lag of) the *j*-th variable equals to

$$\sigma_i^2/\sigma_j^2, \quad \text{(A6)}$$

where σ_i^2 is the estimated standard error from the univariate *AR(4)* model of variable i.

The prior on the error covariance matrix $\bar{\Sigma}^{(r)}$ is distributed as the inverse Wishart distribution. The scale matrix is such that its diagonal elements are equal to the estimates of the error variances in the *AR(4)* models of the respective endogenous variables. The degrees of freedom are equal to $n+1$, suggesting a rather uninformative prior capable of dealing with the fact that variances of the endogenous variables differ by orders of magnitude.

The prior on the unconditional mean of endogenous variables $F^{(r)}$ is distributed normally. In the case of the interest rate and output growth, the means of the prior distribution are based on the estimates from Holston et al. (2017). More precisely, we take their average estimate over the period for which our model is estimated. For inflation, we assume the prior mean to equal two, and the historical average of 0.71 is taken for the spread. The variances of the prior on the steady state values are equal to one. Note that the priors are the same for the steady states in both regimes. The prior distribution on the weights w_i in the definition of the threshold variable in (2) is assumed to be uniform.

Finally, the threshold R is the last parameter for which the prior is formulated, whereby there are two a priori restrictions. First, it is the minimum number of observations in a regime. Second, it is the requirement that the steady state in Regime 1 belongs to Regime 1, i.e. $Y^{TR}\left(F^{(1)}\right) < R$, and/or the steady state in Regime 2 belongs to Regime 2, i.e. $Y^{TR}\left(F^{(2)}\right) \geq R$. These two restrictions are also imposed when taking draws of the threshold during the Gibbs sampling (a detailed discussion follows below). As such, the conditional prior $p\left(R|F^{(1)}, F^{(2)}, w\right)$ is uniform on the set of all values R, which satisfy:

a) for a certain threshold variable (given by w), R implies regimes with at least a given minimum number of observations, and

b) for a given unconditional mean $F^{(1)}$ and $F^{(2)}$ and for the threshold variable defined by w, R is such that $Y^{TR}\left(F^{(1)}\right) < R$ or $Y^{TR}\left(F^{(2)}\right) \geq R$.

The joint prior on the vector of parameters $\boldsymbol{\theta}$ is formulated as follows:

$$p(\boldsymbol{\theta}) \propto \left[\prod_{r=1}^{2} p(\beta^{(r)})\, p(\bar{\Sigma}^{(r)}) p(F^{(r)})\right] p(R|F^{(1)}, F^{(2)}, w) p(w). \tag{A7}$$

A.3 Conditional Posteriors and the Gibbs Sampler

Following the Bayes theorem, combining the likelihood in (A5) and the joint prior in (A7) yields a joint posterior. The joint posterior distribution is then simulated by taking draws from the conditional posteriors.

Let $\boldsymbol{\theta}_{-parameter}$ denote the parameter vector excluding a particular parameter. For example, $\boldsymbol{\theta}_{-R}$ contains all model parameters other than R: $\boldsymbol{\theta}_{-R} \equiv [\beta^{(1)}, \beta^{(2)}, F^{(1)}, F^{(2)}, w, \bar{\Sigma}^{(1)}, \bar{\Sigma}^{(2)}]$. Let us now describe the steps in the simulation procedure and discuss the conditional posteriors.

1) The conditional posterior of the vectors of AR parameters $\beta^{(1)}$ and $\beta^{(2)}$ is distributed normally:

$$p\left(\beta^{(r)} | \bar{y}, \boldsymbol{\theta}_{-\beta^{(r)}}\right) \propto N(\beta_{POST}^{(r)}, \Omega_{POST}^{(r)}), \tag{A8}$$

where

$$\beta_{POST}^{(r)} = \Omega_{POST}^{(r)} \left[\left(\Omega_{PRIOR}^{(r)}\right)^{-1} \beta_{PRIOR}^{(r)} + \bar{X}_{dem}^{(r)\,\prime} \left(\bar{\Sigma}^{(r)}\right)^{-1} \bar{X}_{dem}^{(r)} \beta_{OLS}^{(r)}\right], \tag{A9}$$

and

$$\Omega_{POST}^{(r)} = \left[\left(\Omega_{PRIOR}^{(r)}\right)^{-1} + \bar{X}_{dem}^{(r)\,\prime} \left(\bar{\Sigma}^{(r)}\right)^{-1} \bar{X}_{dem}^{(r)}\right]^{-1}. \tag{A10}$$

The variable $\bar{X}_{dem}^{(r)}$ denotes demeaned data matrices and $\beta_{OLS}^{(r)}$ is a standard OLS estimate of the vector $\beta^{(r)}$ estimated on the demeaned data. The demeaning of the data is due to the fact that the unconditional means of endogenous variables are treated separately in the estimation procedure.

For each draw from the conditional posterior, a stability check is carried out, i.e. the eigenvalues related to the regime-specific VAR are compared to unity. In the case of an eigenvalue larger than one, the system is unstable and another draw is taken from the conditional posterior. The maximum of tries is set to 10,000. If neither $\beta^{(1)}$ nor $\beta^{(2)}$ ensuring a stable VAR is found, the draws from other conditionals based on this 'unstable' draw are discarded. It turns out that in all estimations (with only one exception) the ratio of such $\beta^{(1)}$ or $\beta^{(2)}$ does not exceed 30%, and thus the stability requirement should not affect the efficiency of the sampler significantly.

2) The conditional posterior of the error covariance matrix $\bar{\Sigma}^{(1)}$ and $\bar{\Sigma}^{(r)}$ is distributed as inverse Wishart:

$$p(\Sigma^{(r)} | \bar{y}, \boldsymbol{\theta}_{-\Sigma^{(r)}}) \propto iW(S_{POST}, DoF_{PRIOR} + t^{(r)}), \tag{A11}$$

where

$$S_{POST} = S_{PRIOR} + (\bar{y}_{dem}^{(r)} - \bar{X}_{dem}^{(r)} \beta^{(r)})'(\bar{y}_{dem}^{(r)} - \bar{X}_{dem}^{(r)} \beta^{(r)}). \tag{A12}$$

3) The conditional posterior of the vector of unconditional means $F^{(r)}$ is distributed normally:

$$p\left(F^{(r)}|\bar{y},\boldsymbol{\theta}_{-F^{(r)}}\right) \propto N\left(\varphi_{POST}^{(r)},\Phi_{POST}^{(r)}\right), \tag{A13}$$

where

$$\varphi_{POST}^{(r)} = \Phi_{POST}^{(r)}\left\{\Phi_{POST}^{(r)}{}^{-1}\varphi_{PRIOR} + U^{(r)\prime}vec\left[\left(\bar{\Sigma}^{(r)}\right)^{-1}\left(\bar{y}^{(r)} - \bar{X}^{(r)}\beta^{(r)}\right)'\bar{Z}^{(r)}\right]\right\}, \tag{A14}$$

and

$$\Phi_{POST}^{(r)} = \left[\Phi_{POST}^{(r)}{}^{-1} + U^{(r)\prime}\left(\bar{Z}^{(r)\prime}\bar{Z}^{(r)} \otimes \left(\bar{\Sigma}^{(r)}\right)^{-1}\right)U^{(r)}\right]^{-1}, \tag{A15}$$

such that

$$U^{(r)} \equiv \begin{pmatrix} I_4 \\ A_1^{(r)} \\ \vdots \\ A_p^{(r)} \end{pmatrix}.$$

The data matrices $\bar{y}^{(r)}$ and $\bar{X}^{(r)}$ are not demeaned. The last subset of the parameter vector $\boldsymbol{\theta}$ consists of the weights constituting the threshold variable and the threshold value r. Both types of parameters are treated as discrete variables.

4) The space for weights $[w_1,\dots,w_4]$ is discretized such that we work with all 4-tuples, with each element taking values from $\{0,0.1,\dots,0.9,1\}$ and adding up to one. The likelihood of data is then computed for each 4-tuple and a random draw from the multinomial distribution with probabilities given by the likelihood.[18]

5) Finally, the conditional posterior of the threshold is affected by the prior conditional on the vector of weights w and by the unconditional means $F^{(1)}$ and $F^{(2)}$. The set of possible values of the threshold is restricted such that:

a) at least a given minimum number of observations is in each regime, and

b) at least one regime-specific vector of the unconditional means is in its own regime.

From an econometric point of view, in imposing a minimal number of observations we face a trade-off. On the one hand, rare events may be represented by a few observations only and their solid modelling is important for an accurate probabilistic assessment of the output growth distribution tails. On the other hand, some minimal number of observations is necessary to estimate our relatively heavily parameterized model in order to avoid the estimation results being driven by the assumed priors. Furthermore, imposing stability on both regime-specific VARs requires regimes with a higher number of observations – otherwise the stability condition may not always be satisfied. We assume that a regime contains at least 28 observations out of the 120 total

[18] The implementation of the random draw from the multinomial distribution is influenced by the fact that log-likelihood and not likelihood of data (given a 4-tuple of weights) are computed. Then we exploit the fact that a log of uniform random value is equivalent to a negative of an exponential random draw, and we can therefore stick to the log space when implementing random draws from the multinomial distribution.

observations, i.e. 7 out of 30 years. The reason is that this seven-year minimum is close to the average length of the US business cycle.

Next, to avoid unrealistic oscillating patterns of macroeconomic variables that would distort our risk measures, the threshold variable applied on values of the unconditional mean should yield a value that implies that the system is not oscillating in the steady state.

Then all eligible values of the threshold are evaluated in terms of the likelihood, and a random draw is taken from the multinomial distribution with probabilities related to the likelihoods. The stability check on $\beta^{(1)}$, $\beta^{(2)}$ and the conditions in point 5) are not sufficient conditions for the stability of a threshold VAR. We therefore check each simulated path of output growth and exclude those suggesting exploding patterns that exceed 400 in absolute value, i.e. those that describe the fall of the economy's production between two quarters as being equal to what the economy produced in the previous period. In this way, not more than 4% of the simulated paths (and the corresponding TVARs) are excluded.

The Gibbs sampler involves repeating steps 1)–5). The initial values include (i) the OLS estimates for the AR parameters, (ii) the degrees-of-freedom-adjusted estimate of the error covariance matrices in the regimes for the error covariance matrix, (iii) the means of the prior distributions for the unconditional means, (iv) a random draw from the uniform distribution for the weights, and finally (v) a random draw from the uniform distribution over all threshold values which satisfy the two above-mentioned restrictions.

A.4 Convergence and Efficiency of the Gibbs Sampler

This subsection first examines the efficiency of drawing the threshold and weights, and then presents the standard convergence diagnostics for all other model parameters. Sampling the threshold value and weights in the definition of the threshold variable in (2) are conducted in a novel way – for the procedure see points 4) a 5) in Appendix A.3. The advantage of the procedure is its higher efficiency relative to the standard approach based on the Metropolis step. Let us compare these two approaches in terms of sampling the threshold value R.

The Metropolis step is carried out as follows:

- Proposed value R^* is drawn from the prior distribution.
- The conditional probability of a given threshold value R is given by the following formula:

$$p\left(R|\beta^{(1)},\beta^{(2)},F^{(1)},F^{(2)},w,\bar{\Sigma}^{(1)},\bar{\Sigma}^{(2)}\right)=\left|\bar{\Sigma}^{(1)}\right|^{-\frac{}{2}}\left|\bar{\Sigma}^{(2)}\right|^{-\frac{}{2}}*$$

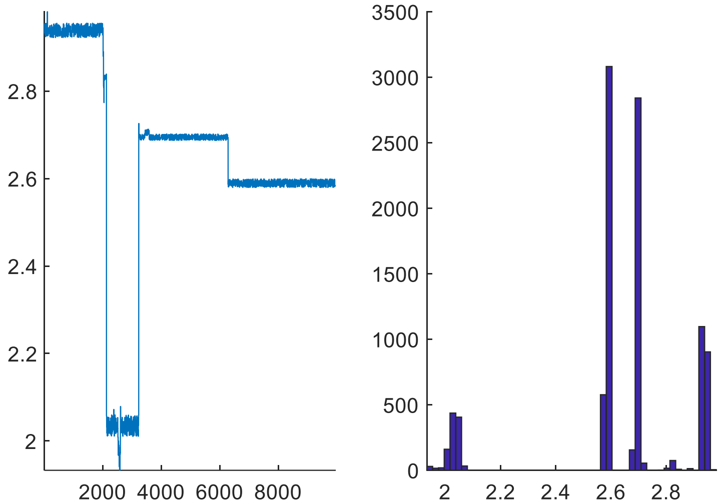
$$*\,exp\left\{-\frac{1}{2}\sum_{r=1}^{2}\left(\bar{y}^{(r)}-\bar{X}^{(r)}\beta^{(r)}-\bar{Z}^{(r)}\delta^{(r)}\right)\left(\bar{\Sigma}^{(r)}\right)^{-1}\left(\bar{y}^{(r)}-\bar{X}^{(r)}\beta^{(r)}-\bar{Z}^{(r)}\delta^{(r)}\right)\right\}. \qquad \text{(A16)}$$

  The conditional probability (A16) of the proposed value R^* is compared with the conditional probability of the R from the previous iteration.

- The proposed value is accepted with probability $\min\{1,p(R^*|\dots)/p(R|\dots)\}$, where $p(.|\dots)$ is defined in (A16). Technically, the procedure is employed by taking the difference between the logs of the two conditional probabilities above. If it is greater than the log of a draw from a standard uniform distribution, the proposed value is accepted.

Figure A1 presents a chain of 10,000 draws of the threshold value and a histogram corresponding to the chain using the Metropolis step. Figure A2 shows the analogous output using our approach, i.e. the multinomial distribution for taking draws of the threshold value.[19] The figures demonstrate that our approach of discretizing the parameter space and using the multinomial distribution leads to a more efficient estimation. A common way to assess this formally is the inefficiency factor, which is defined as $1 + 2\sum_{s=1}^{\infty} \rho_s$, where ρ_s represents the sample autocorrelation at length s based on the sampled draws of a parameter.[20] Based on this measure, the improvement in the efficiency of the Gibbs sampler when moving from the conventional Metropolis step to our multinomial distribution is substantial; the inefficiency factor declines from 733 to 303. This is also apparent from the comparison of Figures A1 and A2.

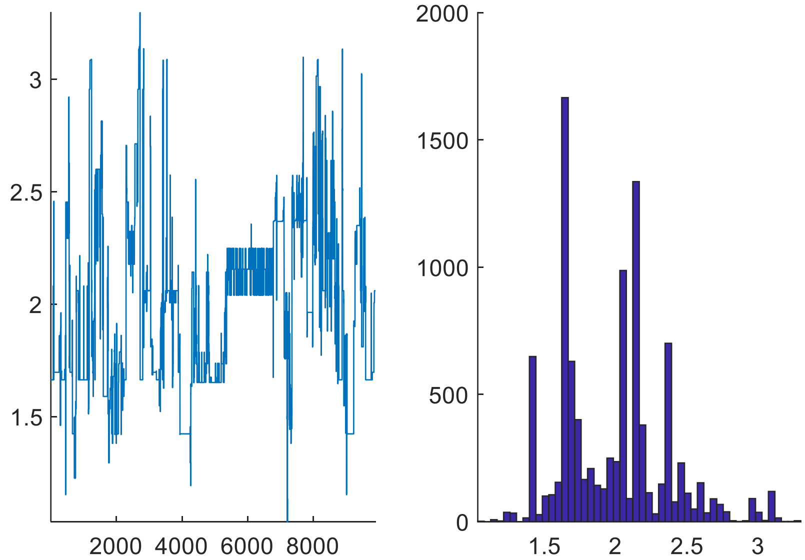
Figure A1: Gibbs Sampler – Draws of the Threshold Value in the Algorithm Using the Conventional Metropolis Step



[19] We employ a uniform conditional prior for the threshold value in both approaches in Figures A1 and A2.

[20] We use the first 500 sample autocorrelations to compute the inefficiency factor.

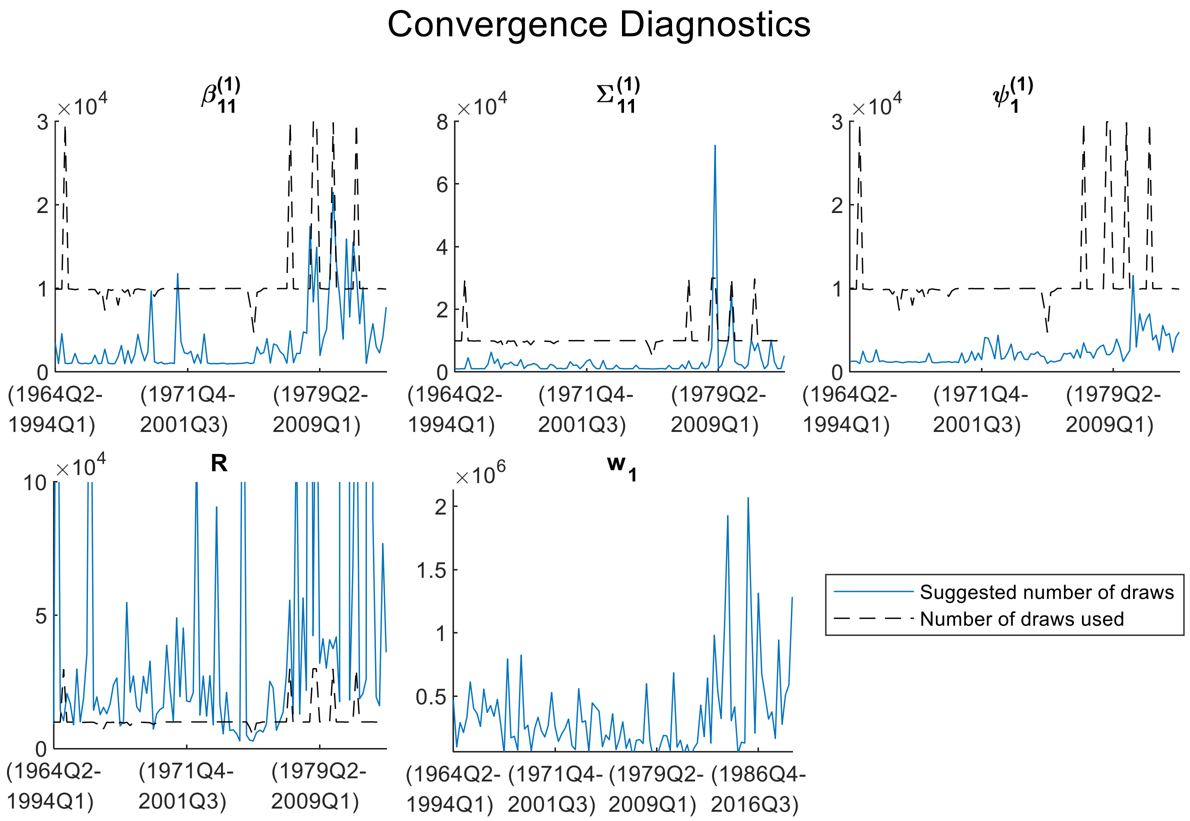
Figure A2: The Gibbs Sampler – Draws of the Threshold Value in the Algorithm Using our Approach with the Multinomial Distribution



The convergence diagnostics for the rest of the parameters is presented in Figure A3. The convergence measure is based on Raftery and Lewis (1992). It suggests how many draws should be taken from the conditional posteriors within the Gibbs sampler to obtain a stationary joint distribution.[21]

[21] The usual diagnostics parameters are used: for the 0.025th and 0.975th quantiles of a marginal posterior distribution, an accuracy of 0.025 with a probability of 0.95 must be achieved.

Figure A3: Convergence Diagnostics of the Selected Model Parameters Based on Raftery and Lewis (1992)



Note: The dashed line indicates the number of Gibbs iterations used in the estimation of the model.

The convergence diagnostics suggest a higher number of draws for some parameters and estimation windows. We thus increase the number of iterations to 90,000 for some estimation windows and take each 3rd iteration to deal with chain autocorrelation. In regards to the threshold and the first element of the weights vector, the convergence diagnostics suggest that an even higher number of draws may be optimal for some estimation windows, but this is not implemented to ensure the time feasibility of the rolling window exercise.

A.5 Estimation with Correlated Variables in the Threshold Variable

One of the estimation issues relates to the correlation of endogenous variables constituting the threshold variable y_t^{TR}. It could in principle raise concerns about the efficiency of the Gibbs sampling. Strongly correlated endogenous variables could make the estimation of weights in the threshold variable problematic because the likelihood differs only marginally depending on which of the correlated variables drives the threshold variable. Moreover, negatively correlated endogenous variables could lead to switching between the regime labels.

Consider, for instance, a simplified model with two endogenous variables only and with the threshold variable without smoothing. In this case, the threshold variable equals $y_t^{TR} = w_1 y_{1t} + (1 - w_1) y_{2t}$. For illustration, focus on the polar case of perfect negative correlation between the two endogenous variables $y_{2t} = -K y_{1t}$, for some $K > 0$. In such special cases, for a given vector of weights and the threshold value, Regime 1 is defined as the set of periods satisfying the following condition:

$y_t^{TR} = w_1 y_{1t} + (1 - w_1) y_{2t} < R.$

Then for $w_1 = 0$ and some R we get Regime 1 if

$y_t^{TR} = y_{2t} = -K y_{1t} < R$ i. e. $-y_{1t} < (1/K)R$,

and for $w_1 = 1$ and some $\tilde{R}$ we get Regime 1 if

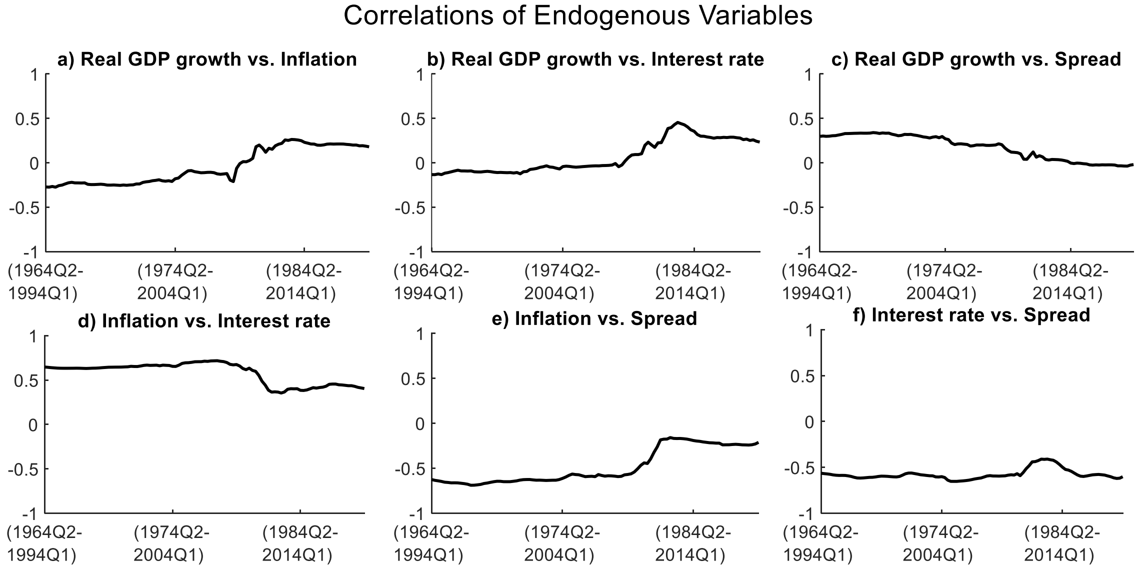
$y_t^{TR} = y_{1t} < \tilde{R}.$

Intuitively, we have two different vectors of weights $[0,1]$ and $[1,0]$ and two different threshold values R and $\tilde{R}$, which imply exactly the same regimes but with the opposite labels. In particular, Regime 1 is constituted by periods such that $-y_{1t} < (1/K)R$ in the first case, and $y_{1t} < \tilde{R}$ in the second. The estimation procedure views these two cases as equivalent in terms of the likelihood. This may lead to a switch of regimes between different estimation windows and also within the Gibbs sampler, affecting its efficiency in an undesirable way.

In the type of modeling exercise we perform, such a problem can occur in principle because the spread is strongly negatively correlated with inflation and the interest rate for some estimation windows (Figure A4), and at the same time the interest rate dominates the threshold variable (Figure B3). Our solution is to simply work with the *negative* of the spread in the vector of the endogenous variables. The only two exceptions are the estimation windows 1987Q1-2016Q4 and 1989Q1-2018Q4, for which the Gibbs sampler converges much more efficiently when the positive of the spread is used.

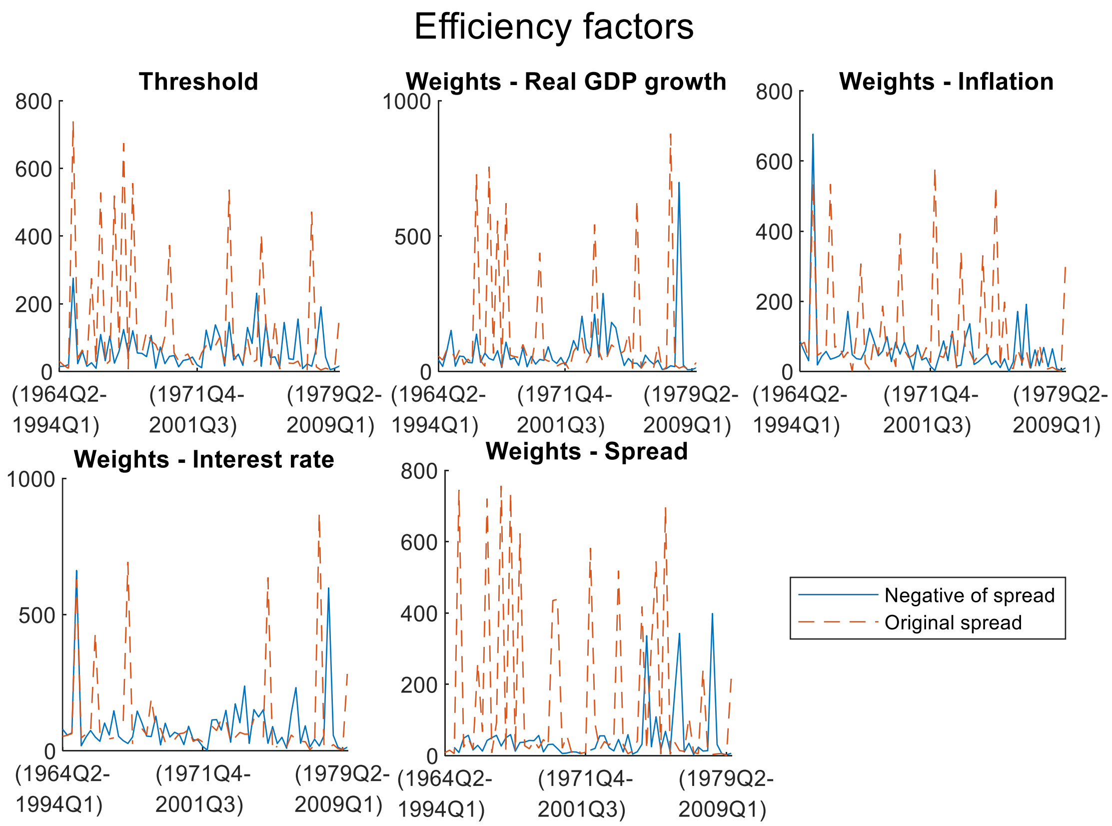
Figure A5 shows inefficiency factors for the estimation windows for which we use the negative of the spread, and it compares them to the specification featuring the original spread. In many instances our approach to correlated variables in the threshold variable improves efficiency substantially.

Figure A4: Correlation Coefficients of the Endogenous Variables



Note: The x-axis indicates the range of quarters of the estimation window.

Figure A5: Inefficiency Factors for the Specifications Featuring a Negative of Spread and the Original Spread



Note: The x-axis indicates the range of quarters of the estimation window.

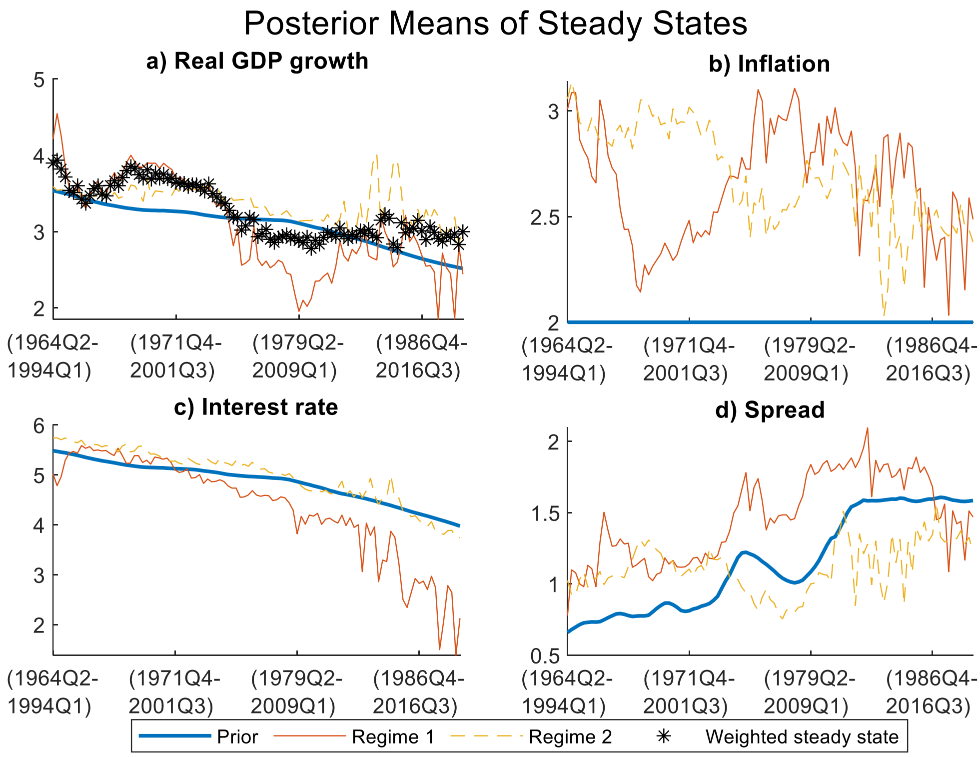
Appendix B: Additional Results and Procedures

In this appendix, we present additional results and procedures related to: (i) the estimation of the threshold VAR model (Section B.1), (ii) the counterfactuals of the long-run output growth distribution moments (Section B.2), (iii) the bootstrap procedure for the confidence intervals in the equation for conditional moments (Section B.3), (iv) the normality tests of the short-run and long-run output growth distribution (Section B.4), and (v) the correlation analysis of the shocks' size and the skewness of the unconditional output growth distribution (Section B.5).

B.1 Threshold VAR Model

To understand the nonlinear dynamics between financial conditions and the real economy, a two-regime model is employed. Figures B1 and B2 present the main differences between the regimes. The figures suggest that the regimes in the first part of the series of estimation windows, which include the pre-GFC periods, differ mainly due to different shock volatilities. Once the GFC and post-GFC periods enter the estimation window, the difference between the regimes consists primarily of regime-specific steady states, see Figure B1.

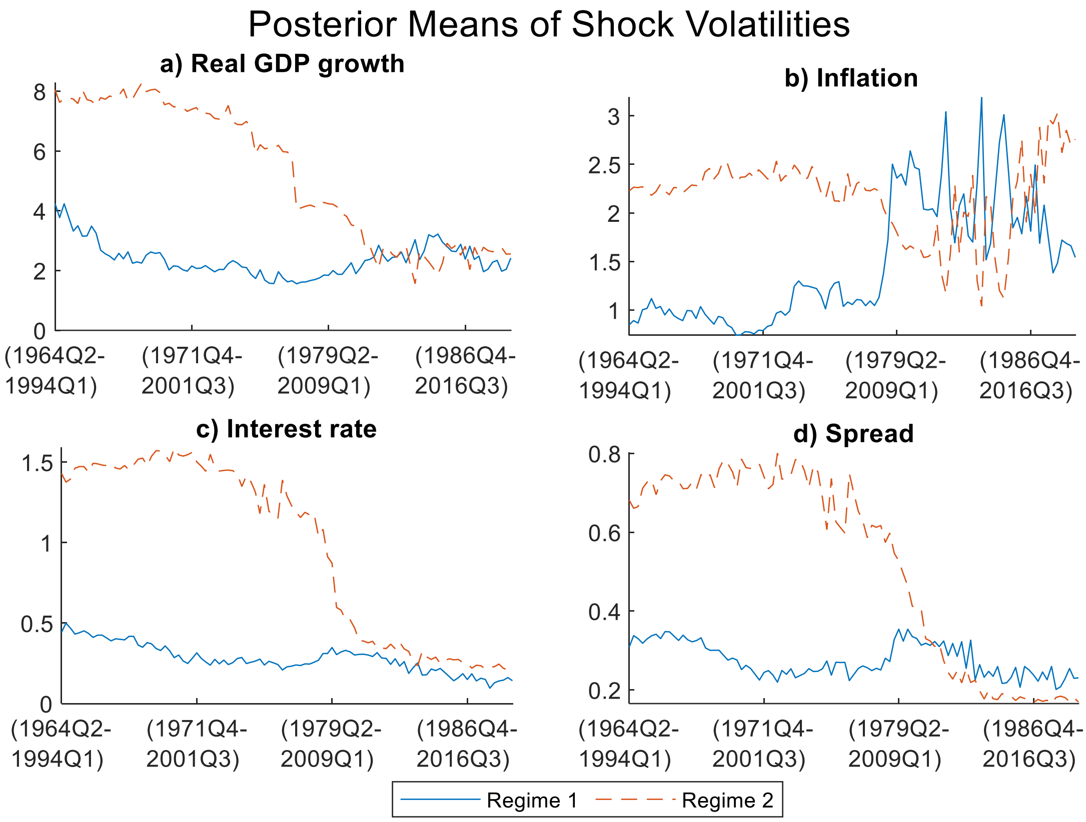
Figure B1: The Mean of the Prior Distribution of the Steady States, the Means of the Posterior Distribution in the Two Regimes, and the Weighted Posterior Mean of Long-run Output Growth with Weights Equal to the Probability of the Regime in the Long Run



Note: The x-axis indicates the range of quarters of the estimation window.

The different output growth steady states shown in panel a) of Figure B1 imply the bimodality of the long-run output growth distribution. This is in line with Adrian et al. (2021), who found the bimodality of the output growth distribution to be a feature related to financially turbulent times, whereas 'normal' times are characterized by a Gaussian output growth distribution.

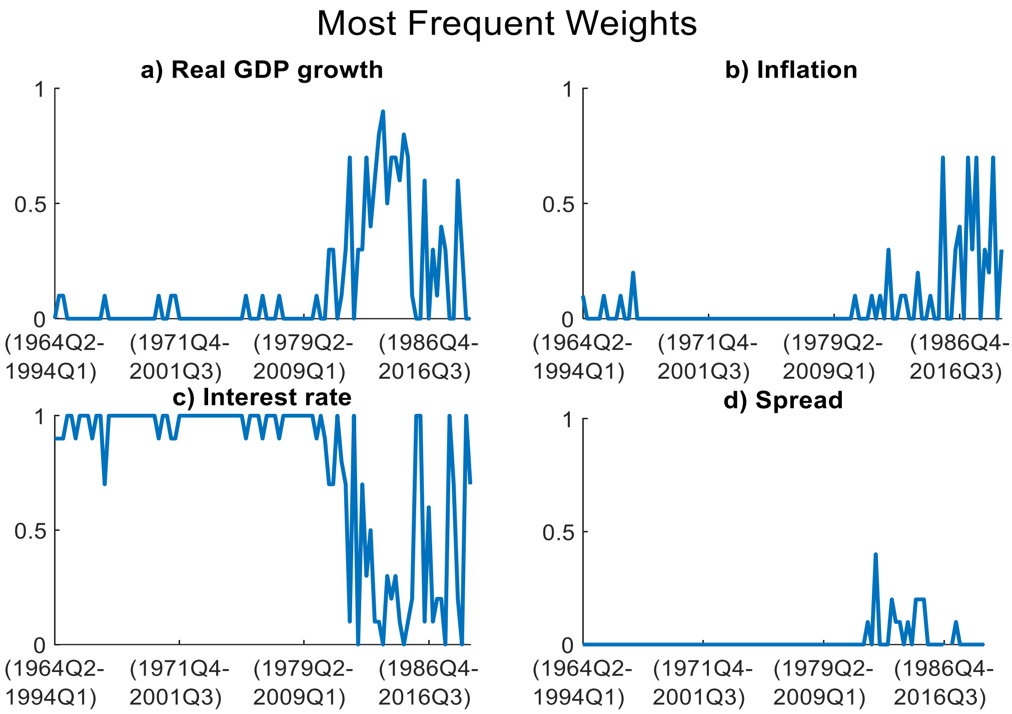
Figure B2: The Posterior Mean of the Shock Volatilities in the Two Regimes



Note: The x-axis indicates the range of quarters of the estimation window.

In addition to the distinct steady states or shock volatilities, the regimes can be interpreted based on the estimated weights in the threshold variable defined in (2). Figure B3 shows the most frequent weights drawn during the Gibbs sampling for each estimation window. The dominant role of the interest rate for the first two-thirds of the sample is apparent. Then real GDP growth and inflation start to drive the regime change. Some role for the spread is found at the end of the sample. This confirms the role of unconventional policy measures captured by the spread variable.

Figure B3: The Most Frequent Weight of Endogenous Variables in the Threshold Variable in the Gibbs Chain

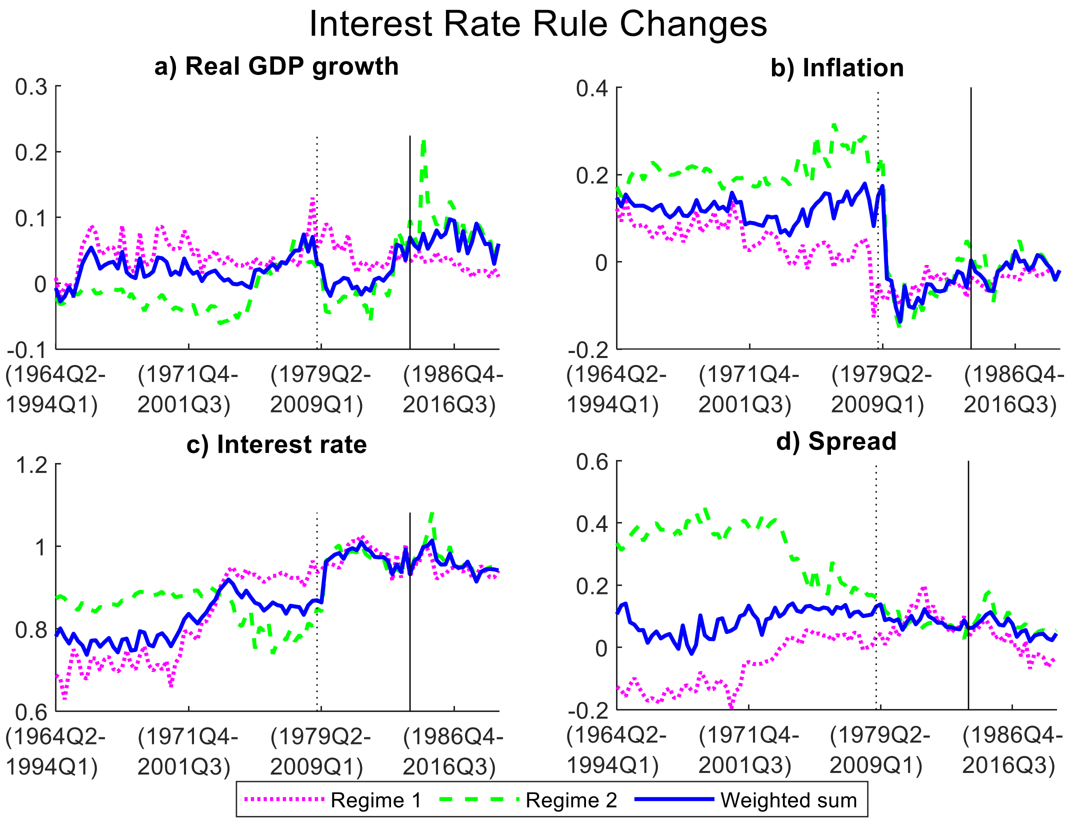


Note: The weights can attain values from the set $\{\mathbf{0}, \mathbf{0.1}, \dots, \mathbf{0.9}, \mathbf{1}\}$ only. For details see Appendix A.3. The x-axis indicates the range of quarters of the estimation window.

While the regime change can be driven by a switch in shock volatilities, the threshold variable is related solely to the levels of the endogenous variables. The shock volatility thus drives the regime change to the extent that it is reflected in the level of endogenous variables. Working with quarterly macroeconomic data representing the real and nominal sides of economy, we do not expect heteroscedasticity of the type observed in financial time series. Instead, changes in shock volatilities of the manner observed during the Great Moderation are captured by our modelling framework. Such changes are indeed reflected in the levels of endogenous variables. Nonetheless, the weights should be interpreted with this caveat in mind.

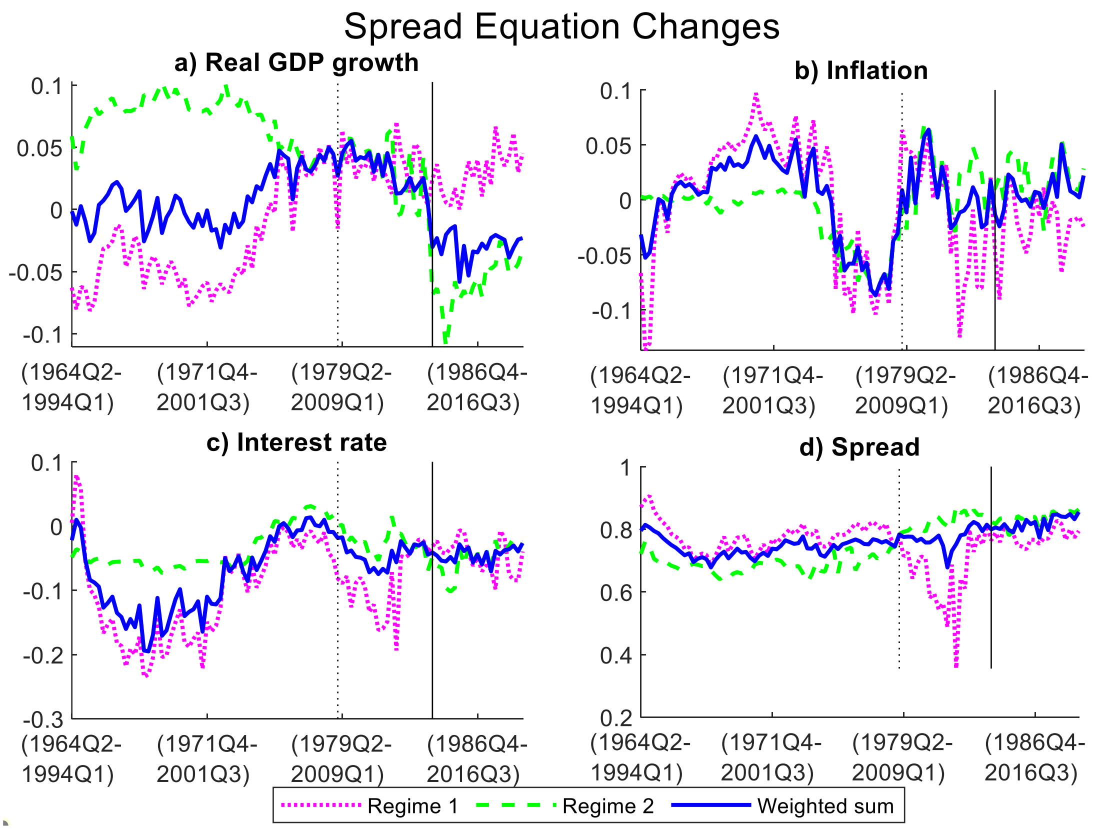
Figures B4, B5, and B6 present the estimated (reduced-form) coefficients at the lagged endogenous variables in the interest rate equation, the spread equation and the inflation equation. More precisely, we take the sum of the estimates at all lags of a given endogenous variable to characterize the overall effect (our interest does not lie in the exact profile). Figures B4 and B5 demonstrate changes in the behavior of the monetary authority after the GFC (the dotted vertical line), i.e. when the GFC enters the estimation windows, and post-2014 when the tail downside risk started to rise (the solid vertical line).

Figure B4: The Sum of the Estimated Coefficients at Lags of the Respective Variable in the Interest Rate Equation, Together with a Weighted Sum over the Two Regimes with Weights Equal to the Probability of the Regime in the Steady State



Note: The x-axis indicates the range of quarters of the estimation window. The dotted vertical line indicates the estimation window in which the GFC first enters the estimation (1979Q1-2008Q4). The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

Figure B5: The Sum of the Estimated Coefficients at Lags of the Respective Variable in the Spread Equation, Together with a Weighted Sum over the two Regimes with Weights Equal to the Probability of the Regime in the Steady State



Note: The x-axis indicates the range of quarters of the estimation window. The dotted vertical line indicates the estimation window in which the GFC first enters the estimation (1979Q1-2008Q4). The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

Regarding the change in the behavior of the monetary authority during and after the GFC, it turns out that there is a break in the AR parameters at lagged inflation from positive values to values close to zero starting with the window of 1979Q2-2009Q1 (Figure B4, panel b). Such change can be observed in both regimes and also if we summarize the information across the regimes by a weighted sum of the AR parameters with weights equal to the probability of each regime in the long run (the solid curve). Furthermore, the role of output growth in the interest rate equation declined to zero when the GFC entered the estimation windows, and then became important when the post-2014Q1 entered the estimation windows. The zero effect of lagged output growth on the interest rate can also be observed at the very beginning of the series of estimation windows when the mid-1960s and 1970s are included.

The link between the spread and lagged output growth changed qualitatively as well (see Figure B5, panel a). The positive reduced-form relationship estimated for the windows containing the GFC switched to a negative one when the post-2014Q1 period entered the windows.

Note that the change in the AR parameters at lagged output growth in the interest rate equation and in the spread equation is not due to substantially different regimes estimated before and after the break date. The regimes continue to be very similar, with Regime 1 covering the crisis periods around 2001 and after 2008. Instead, the change arises from the change in data points, i.e. from dropping observations at the beginning of the window and adding new ones at the end of the window. Some differences are also due to the change in the probability of the regimes.[22]

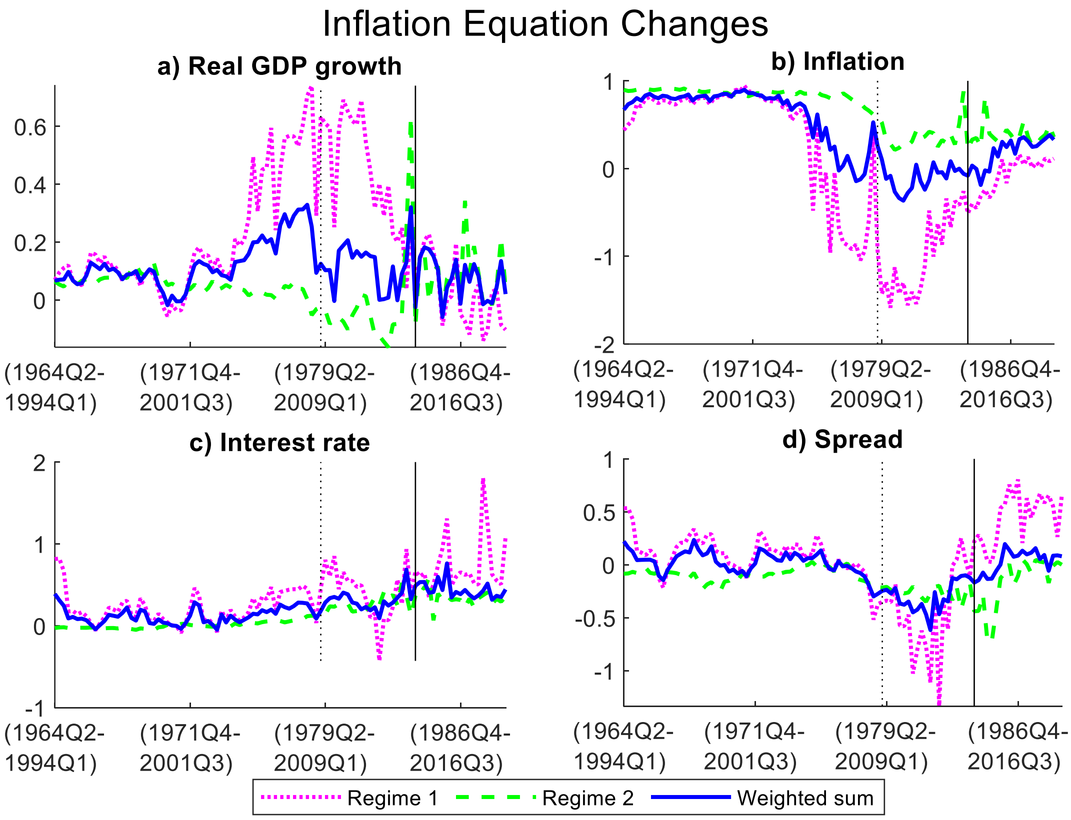
The estimated breaks in the interest rate equation are in accordance with Bianchi (2013), who found two regimes in US monetary policy. His regimes should be compared with our weighted average of the AR parameters. He characterizes his two detected regimes as follows. The "Hawk" regime involves a "strong response to inflation and little concern for output growth", while the "Dove" regime is described by a weak response to inflation. Bianchi (2013) estimated the move from the Hawk to the Dove regime to occur at the end of 2008. Moreover, he also found evidence of the "Dove" regime in the 1970s. Note that Bianchi worked with a structural model that included expectations of regime change. Our results should be interpreted as alternative (reduced-form) evidence supporting his findings.

In terms of Bianchi's (2003) regimes, we find that the "Dove" regime changed again six years later. This occurred when the estimation windows started to include the post-2014 period, i.e. the Fed's normalization of policy through an increase in the Federal Funds rate and QE tapering. The concern for inflation remains negligible (like during the GFC), but the role of output growth has returned to the levels observed before the GFC. As we discuss in Section 0, this change coincides with a break in long-run tail risk, which started to increase post-2014.

A related evaluation of the changes in the macroeconomic dynamics can be found in Liu et al. (2019). The authors focus on the change in dynamics during unconventional monetary policy measures (up to 2011M3), finding a steady decline in inflation persistence. Such finding is supported by our estimates (see Figure B6, panel b). Furthermore, Liu et al. (2019) found that the reaction of the spread to the interest rate shock declines over time. This is confirmed in Figure B5, panel a, in which the effect of the interest rate on the spread approaches zero over time.

[22] Due to space constraints, we are unable to present here the estimated regimes for specific estimation windows. However, they are available upon request.

Figure B6: The Sum of the Estimated Coefficients at Lags of the Respective Variable in the Inflation Equation, Together with a Weighted Sum over the Two Regimes with Weights Equal to the Probability of the Regime in the Steady State

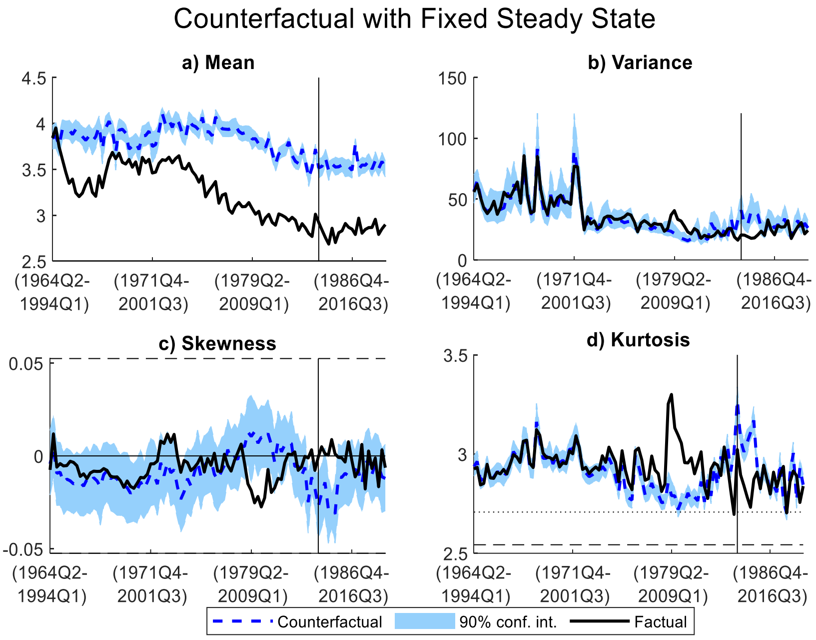


Note: The x-axis indicates the range of quarters of the estimation window. The dotted vertical line indicates the estimation window in which the GFC first enters the estimation (1979Q1-2008Q4). The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

B.2 Distribution Moments of Counterfactuals

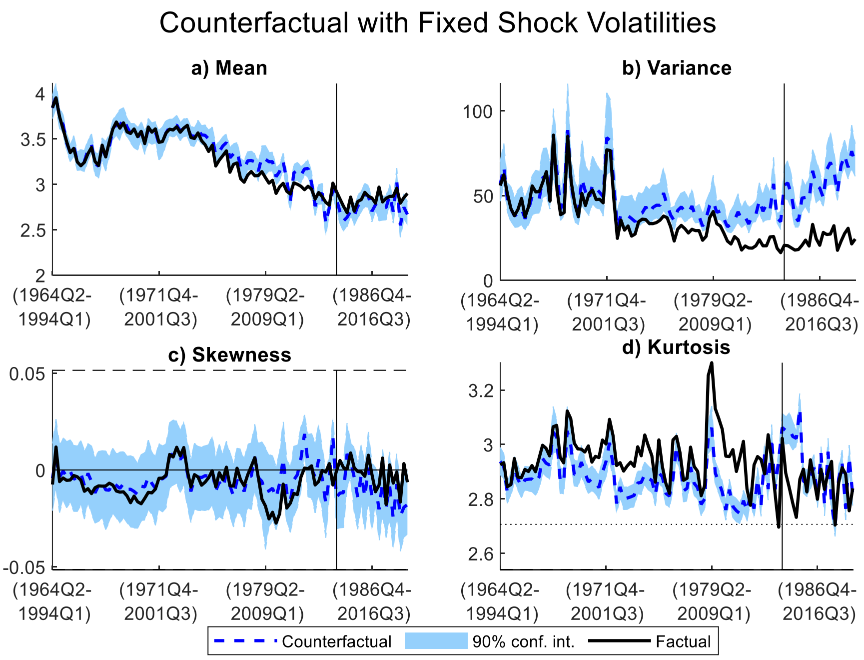
Figures B7, B8 and B9 below present the mean, variance, skewness and kurtosis of the long-run output growth distribution for our three counterfactuals respectively. These were discussed in detail in Section 0.

Figure B7: Moments of the Long-run Output Growth Distribution of the Counterfactual with a Fixed Steady State



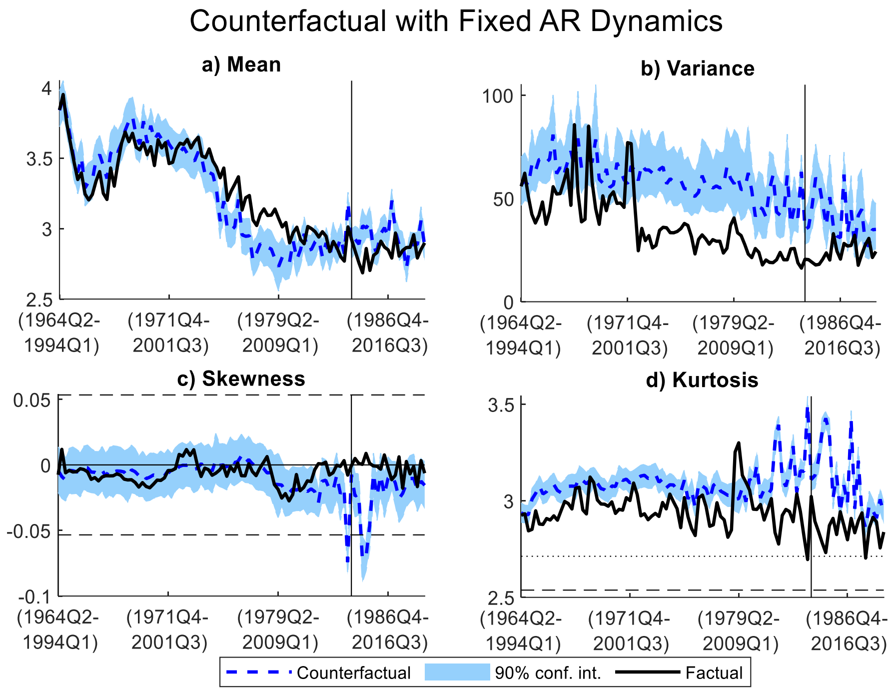
Note: The x-axis indicates the range of quarters of the estimation window. The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

Figure B8: Moments of the Long-run Output Growth Distribution of the Counterfactual with Fixed Shock Volatilities



Note: The x-axis indicates the range of quarters of the estimation window. The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

Figure B9: Moments of the Long-run Output Growth Distribution of the Counterfactual with Fixed AR Parameters



Note: The x-axis indicates the range of quarters of the estimation window. The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

B.3 Confidence Intervals: Model for Conditional Distribution Moments

The model in (4) is estimated by the OLS. Confidence intervals for the parameters of interest $\beta_h^{(m)}$ are constructed employing a bootstrap procedure. To deal with the potential inefficiency of estimates implied by the fact that the dependent variable $\mu_{t+h+4|t+h}^{(m)}$ is generated in the first stage, we take 500 bootstrap samples of $\mu_{t+h+4|t+h}^{(m)}$ by sampling from the set of all simulated forecast paths from the first stage with replacement. Similarly, bootstrap samples are generated for $\mu_{t+4|t}^{(m)}$ to account for the generated regressor. To get confidence intervals for the coefficient $\beta_h^{(m)}$ moving block bootstrap is used. More specifically, for each resampled set of forecast paths, we compute a particular quantile for each t. Then we take blocks of the data matrix (containing both left- and right-hand side variables) of length 4 and a construct a new data matrix of the length of the original data matrix to estimate the model in (4). Then the OLS estimates based on the bootstrap samples are summarized to get the confidence intervals.

B.4 Normality Testing of the Output Growth Distribution

To justify the use of the nonlinear model from the out-of-sample perspective, we conduct two normality tests of the simulated output growth distributions. The first is a one-sample Kolmogorov-Smirnov test with the null hypothesis of the sample being from the standard normal distribution applied on the standardized simulated sample. The second is the Lilliefors test with the null of the sample being generated by a distribution from the family of normal distributions. For the long-run output growth distribution both tests reject the null of normality for all estimation windows. For the output growth distributions four quarters ahead, both tests reject normality in the majority of cases

(62 cases for the Kolmogorov Smirnov test and 92 out of the 101 tests for the Lilliefors test). For the one-quarter-ahead output growth, distribution normality is rejected in less than half the cases for the Kolmogorov-Smirnov test (31 out of 101) and in more than half the cases for the Lilliefors test (64 out of 101 cases).

Such results confirm our focus on the four-quarters-ahead output growth distribution instead of the one-quarter-ahead distribution because non-normality arises over longer horizons. Overall, detected non-normality implies that our threshold VAR model brings new insights that standard (Bayesian) VARs are unable to provide. In addition, taking into account higher moments (such as skewness and kurtosis) of the observed US real GDP growth data, more realistic estimates of output growth distributions and macroeconomic risk can be obtained compared to analyses based on linear models and normal distributions.

B.5 Correlation Analysis of Shocks' Size and Unconditional Output Growth Skewness

To formally demonstrate the link between structural shocks and the asymmetry of the unconditional output growth distribution, we compute the correlation between the standard deviation of the credit spread shock (or the monetary policy shock) and the estimated skewness of the unconditional output growth distribution.

Following the specification of the model in (2) for the excess bond premium, we construct the credit spread shocks as the residuals in the regression of excess bond premium on a constant, contemporaneous Federal Funds rate, contemporaneous output growth, contemporaneous inflation and four lags of the excess bond premium. Since our series of excess bond premium shocks covers the period 1994Q1-2015Q1 only, we then compute the standard deviations of the credit shock on the windows with an increasing size. The first window covers the period 1994Q1 to 1999Q2 (the last quarter before the substantial increase in the spread), whereas the final window covers the whole series of credit shocks (1994Q1-2015Q1). The monetary policy shocks are taken directly as those employed in the analysis presented in Section 6.1. They cover a shorter time span – the final window with the whole series of monetary policy shocks covers 1994Q1-2007Q4.

The correlation analysis reveals a negative relationship between the average size of the credit spread shock and the skewness of the unconditional output growth distribution. As mentioned in the main text, the correlation coefficient is equal to -0.42 and statistically significant at the level of 1% (the confidence interval is bootstrapped based on a bootstrap sample of skewness of size 500). The analogous correlation analysis of the monetary policy shock found no such statistical relationship.

Appendix C: Robustness Checks

This appendix includes several robustness checks related to: (i) the analysis of the distributional effects of structural shocks on the conditional output growth distribution (Sections C.1-C.2), (ii) the credit condition variable employed in the model in (1) (Section C.3), (iii) the choice of the quantile that defines our measure of tail risk (Section C.4), and (iv) the way in which the zero lower bound is imposed in the simulation of predictive distributions (Section C.5).

C.1 Monetary Policy Shock: Alternative Specifications and Shock Measure

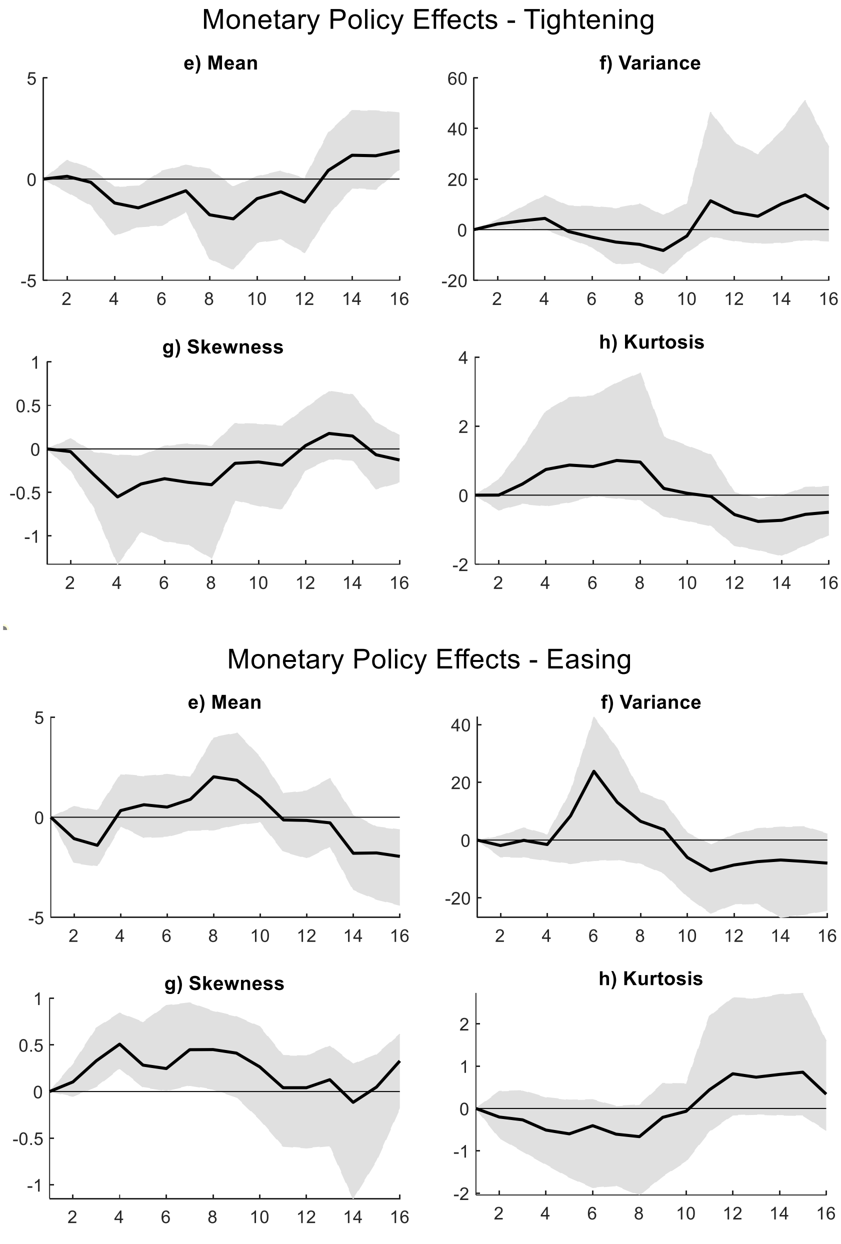
Several specifications of the model for the conditional output growth distribution moments are estimated here to provide further insights and robustness checks. Figures C1, C2, and C3 present the nonlinear variants of the model in (4) separating policy tightening and easing (Figure C1), featuring even moments in logs (Figure C2), and considering the quadratic specification of shocks (Figure C3).

An important aspect of the analysis relates to the exogeneity of monetary policy shocks. The exogeneity of shocks with respect to the conditional mean is guaranteed by the procedure in Romer and Romer (2004). The authors use historical documents and construct a series of intended monetary policy actions, which they confront with real-time macroeconomic data and forecasts to render policy changes orthogonal to the policymakers' information set. They thus filter out the change in the Federal Funds rate that is not related to current and future inflation and real economic activity.

Importantly, Romer and Romer (2004) control for the conditional mean forecasts when estimating unexpected changes in the Federal Funds rate. Despite these measures, endogeneity can still arise in two ways. First, the unexpected change relates to a higher moment of the output growth distribution while, at the same time, the moment affects future output growth moments. Second, higher moments of the future output growth distribution can be affected by a macroeconomic variable not included in the policymakers' information set. If this were to occur, the estimated effect of shocks on variance, skewness and kurtosis would be biased. To shed some light on this question, we conduct a robustness check with differently identified monetary policy shocks. The shocks are taken over from Gertler and Karadi (2015), who look at monetary policy surprises that are orthogonal to both economic and financial variables. The exogeneity with respect to financial variables is crucial because the literature shows financial conditions affect future output growth distributions (Adrian et al., 2019).

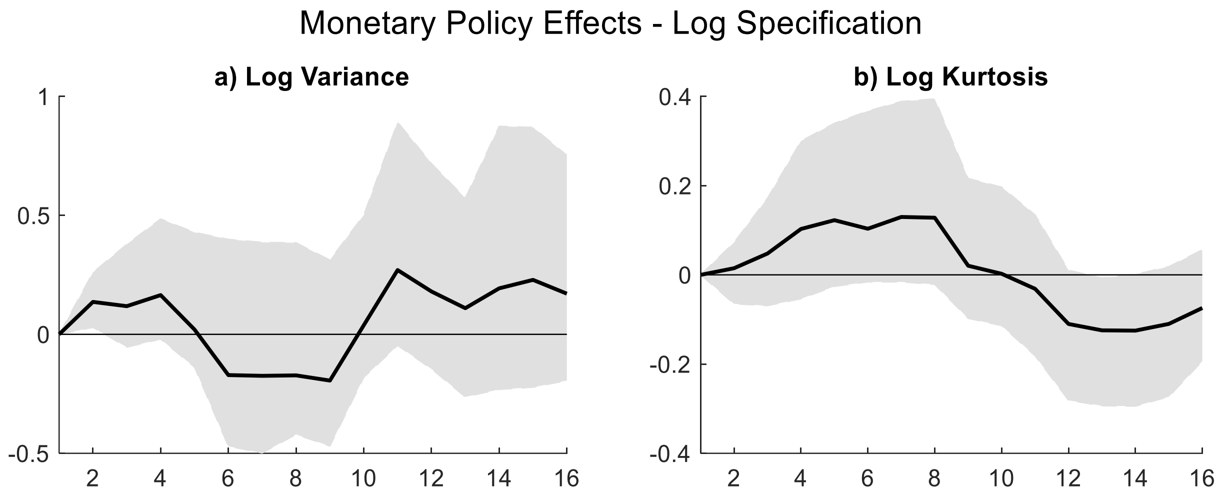
Gertler and Karadi's (2015) identification scheme draws on Federal Funds futures observed on FOMC meeting days and employs a high frequency identification approach. Using monetary policy shocks from Gertler and Karadi (2015) provides similar results to our benchmark shocks taken from Wieland and Yang (2020). In summary, the conditional mean decreases after 2.5 years, the effect on variance is not statistically significant, skewness decreases temporarily and kurtosis is largely unaffected, at least in the medium run (Figure C4).

Figure C1: Response of the Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution to an Unexpected 100-basis-points Monetary Policy Shock



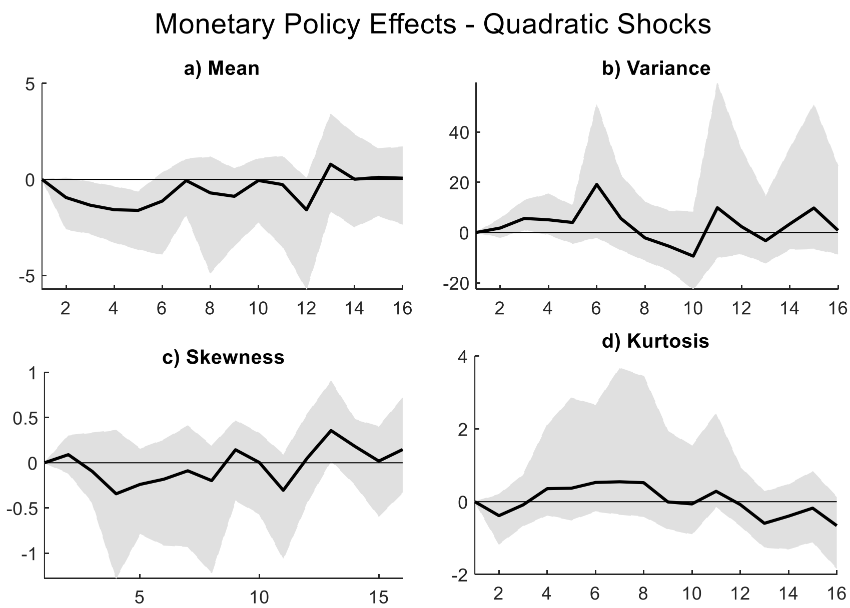
Note: The top panel shows policy tightening, the bottom panel shows policy easing. The shaded areas indicate the 90% confidence intervals. The separate policy shocks are employed in a way that if tightening shocks are considered only, unexpected easing is represented by 0 and vice versa.

Figure C2: Response of the Conditional Variance and Kurtosis for the Specification in Log of Moments; for Monetary Policy Shocks from Romer and Romer (2004) as Updated in Wieland and Yang (2020)



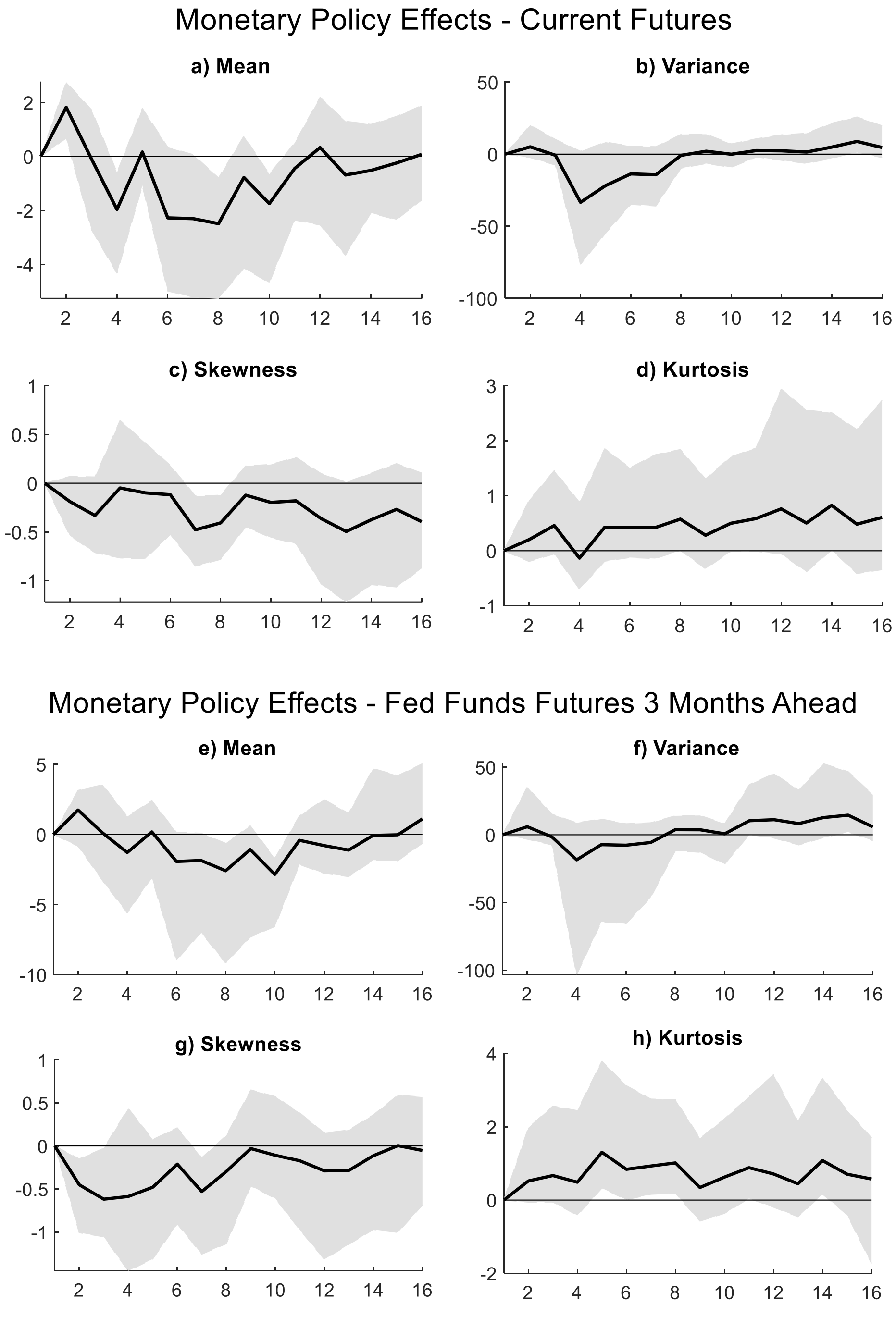
Note: The shaded areas indicate 90% confidence intervals.

Figure C3: Response of the Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution to an Unexpected 100-basis-points Monetary Policy Tightening; for the Specification with Quadratic Monetary Policy Shocks. The Shocks are Taken from Romer and Romer (2004) as Updated in Wieland and Yang (2020).



Note: The shaded areas indicate 90% confidence intervals.

Figure C4: Response of the Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution to an Unexpected 100-basis-points Monetary Policy Tightening with Shocks from Gertler and Karadi (2015)

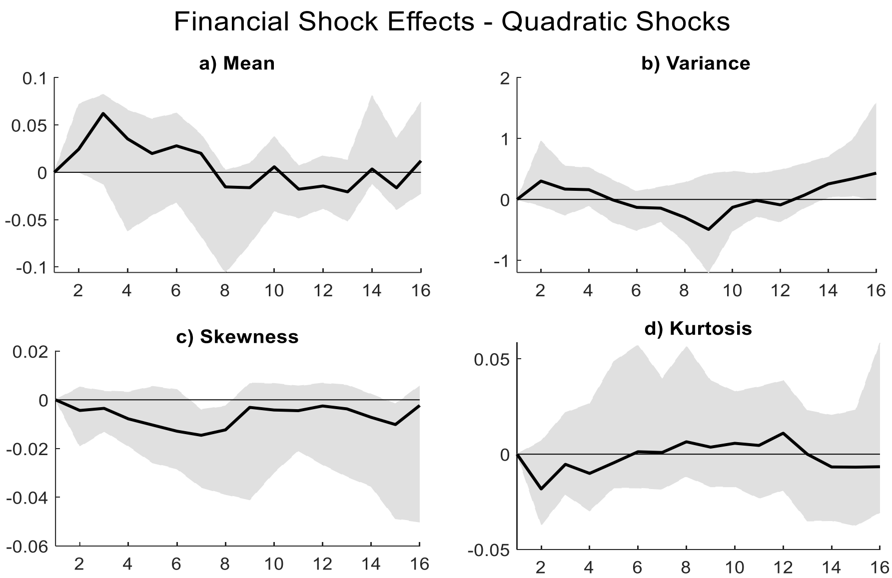


Note: The top panel is based on current futures, the bottom panel is based on the federal funds futures 3 months ahead. The shaded areas indicate the 90% confidence intervals.

C.2 Financial Shock: Alternative Specifications

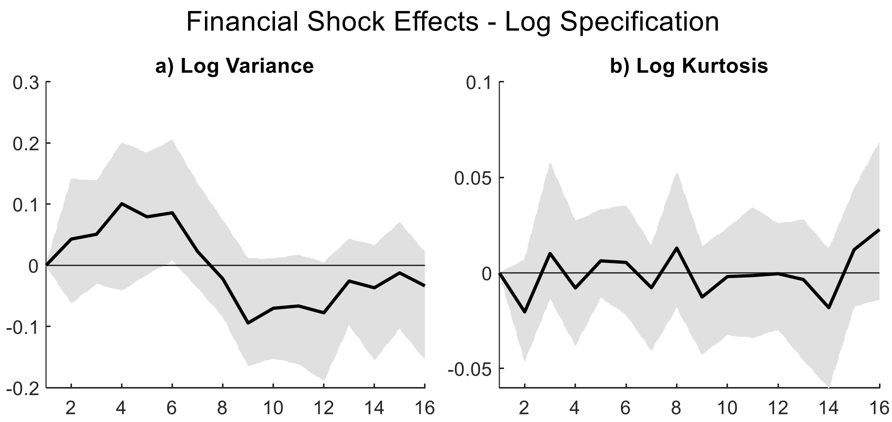
The robustness checks with respect to the specification of the model in (4) confirm the effect of the financial shock on the conditional output growth variance (the quadratic specification of shocks appears in Figure C5); see the discussion in Section 6.2. The specification in logs is presented in Figure C6 and does not provide statistically significant effects.

Figure C5: Response of the Conditional Mean, Variance, Skewness and Kurtosis of the Four-quarters-ahead Output Growth Distribution to an Unexpected 100-basis-points Contractionary Excess Bond Premium in a Specification with Quadratic Shocks



Note: The shaded areas indicate 90% confidence intervals.

Figure C6: Response of the Conditional Variance and Kurtosis for the Specification in the Log of Moments for Excess Bond Premium Shocks

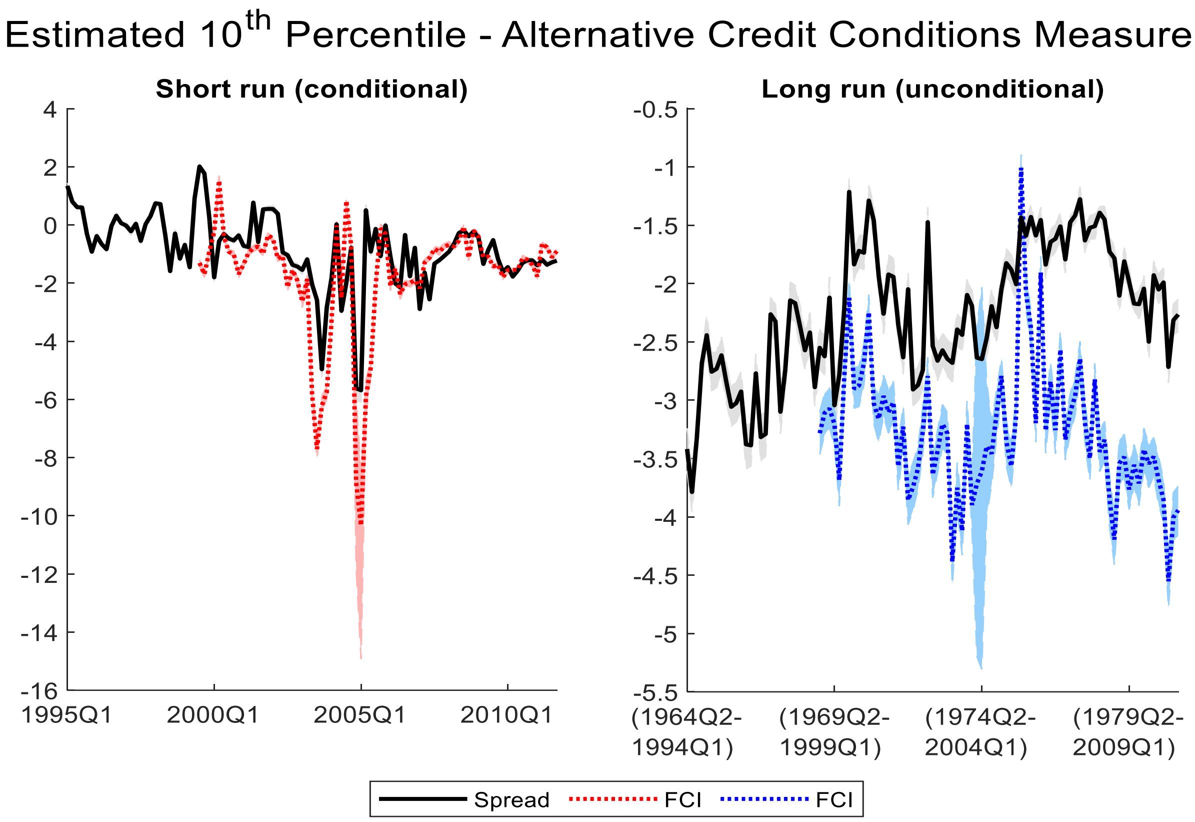


Note: The dashed lines indicate 90% confidence intervals.

C.3 Financial Cycle Indicator Replacing the Term Spread

Borio et al. (2019) examine the importance of the financial cycle for the evolution of US real economic activity in comparison with the term spread, which is employed in our benchmark specification. The authors define the term spread as the difference between the 10-year government bond rate and the 3-month money market rate, whereas our definition includes the Federal Funds rate. They compare various financial cycle measures to the term spread to assess the out-of-sample ability to predict recessions, finding them to be very useful in terms of their in-sample and out-of-sample fit. Therefore, we conduct a robustness check with the credit component of the Chicago Fed National Financial Conditions, see Figure C7. Note that the financial conditions subindex is available from 1971Q1, so the first estimation window of this alternative specification is 1971Q1-2000Q4.

Figure C7: The 10th Percentile of the Four-quarters-ahead (Left Panel) and Long-run (Right Panel) Output Growth Distribution from the Model Specification Featuring the Term Spread and the Credit Component of the Financial Conditions Index (FCI)



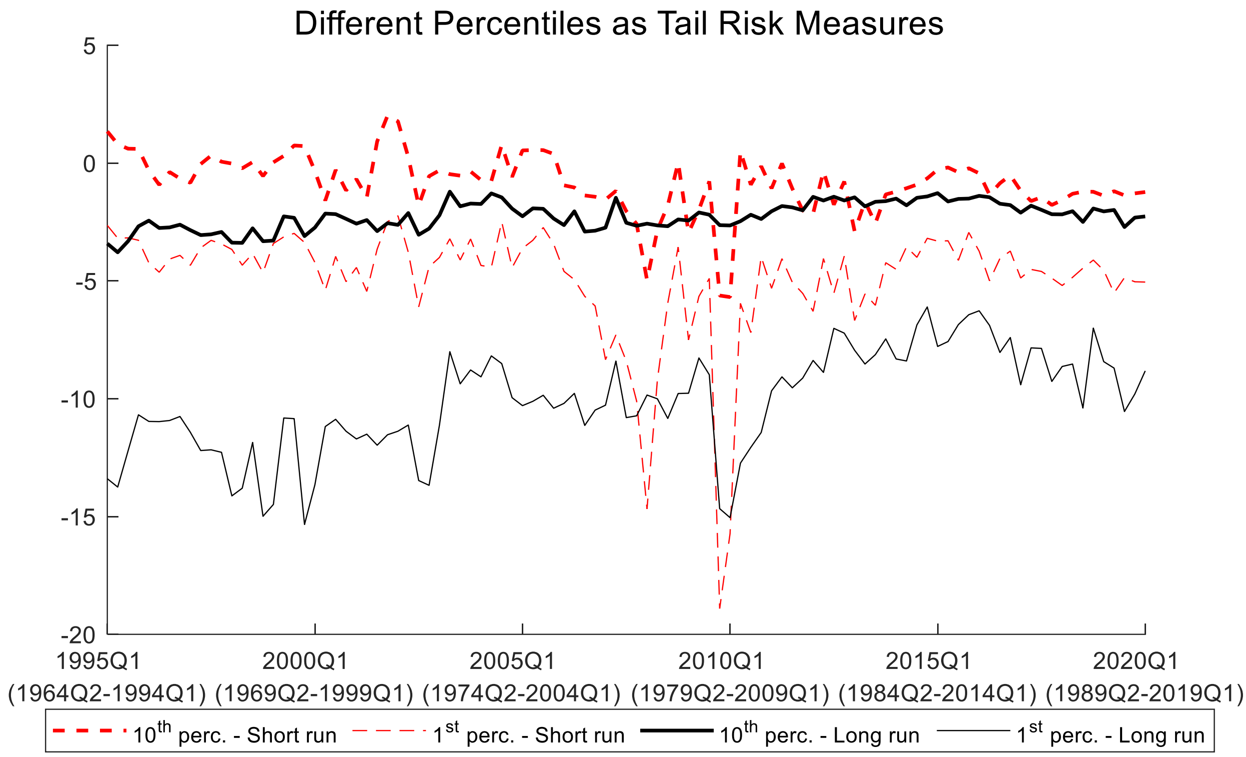
Note: The x-axis indicates the quarter for which the percentiles of conditional output growth distribution are estimated. The periods in parenthesis indicate the windows used to estimate the long-run output growth percentiles. The dashed lines indicate 90% confidence intervals.

Figure C7 demonstrates that the short-run downside tail risk is very similar in both specifications. In terms of the long run, the 10th percentile of the output growth distribution is shifted downwards in the FCI specification. This is a consequence of higher estimation uncertainty related to the model containing the FCI because it translates into a higher variance of the unconditional output growth distribution. Indeed, comparing the distance between the 10th and the 90th percentiles of the unconditional distribution between the two specifications reveals a relatively constant difference over the whole series of estimation windows equal to almost 2 percentage points. The fact that the profile of the 10th percentile in the FCI specification remains comparable to the benchmark specification provides further support for our modelling approach.

C.4 Estimate of the 1st Percentile of the US Output Growth Distribution

Figure C8 presents the 1st and 10th percentiles of both the conditional and unconditional output growth distributions. It demonstrates that focusing on the latter as a measure of downside risk is warranted; moving towards lower quantiles does not offer additional insights. The fact that the 1st percentile is more variable than the 10th (both short-run and long-run) is to be expected.

Figure C8: The 1st and 10th Percentile of the Four-quarters-ahead and Long-run Output Growth Distribution



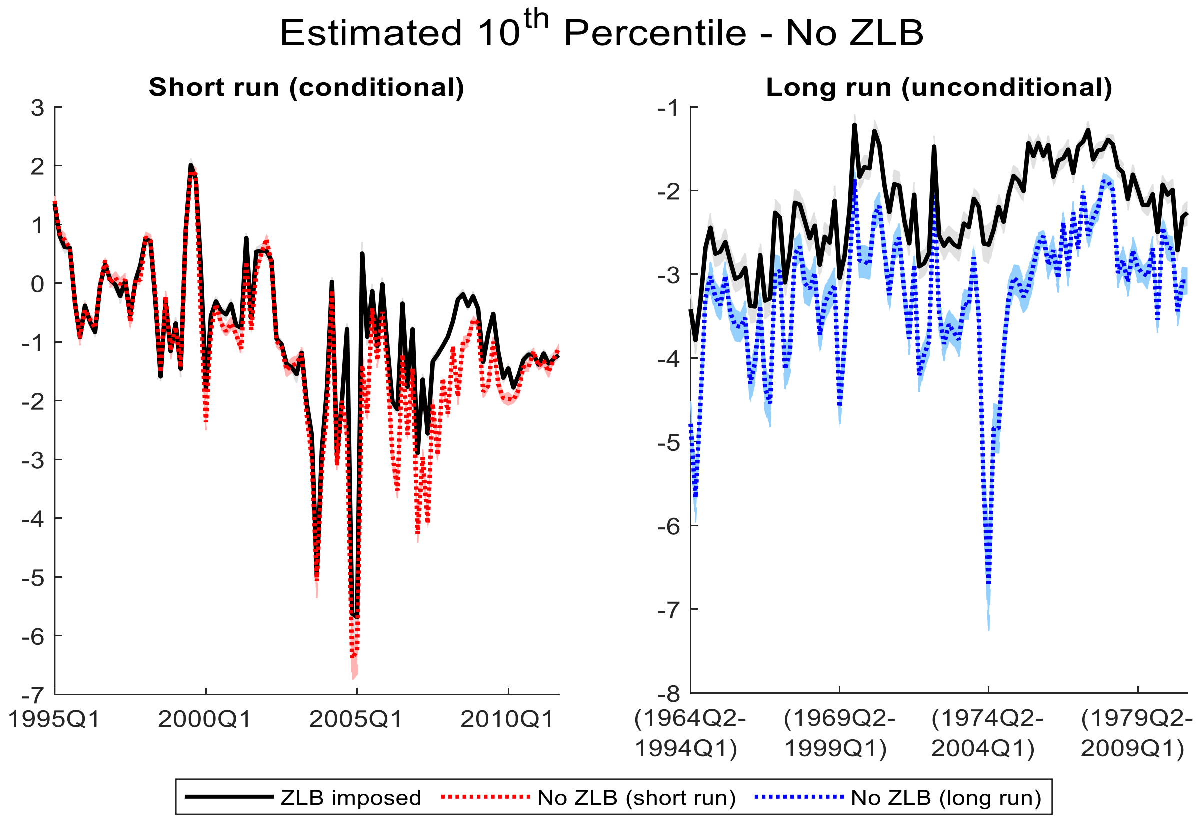
Note: The x-axis indicates the quarter for which the percentiles of the conditional output growth distribution are estimated. The periods in parenthesis indicate the windows used to estimate the long-run output growth percentiles.

C.5 The Effect of (Not) Imposing the ZLB in the Simulation of Predictive Distributions

During the simulation of predictive distributions in the main text, the zero lower bound was implemented in the most natural way by imposing a zero interest rate whenever its path tended to fall below zero. The issue of the ZLB is revisited in Figures C9 and C10, which provide the simulation results without the ZLB being imposed. It is apparent that the main features of the conditional and unconditional distributions remain intact, and so do our key findings in the main text.

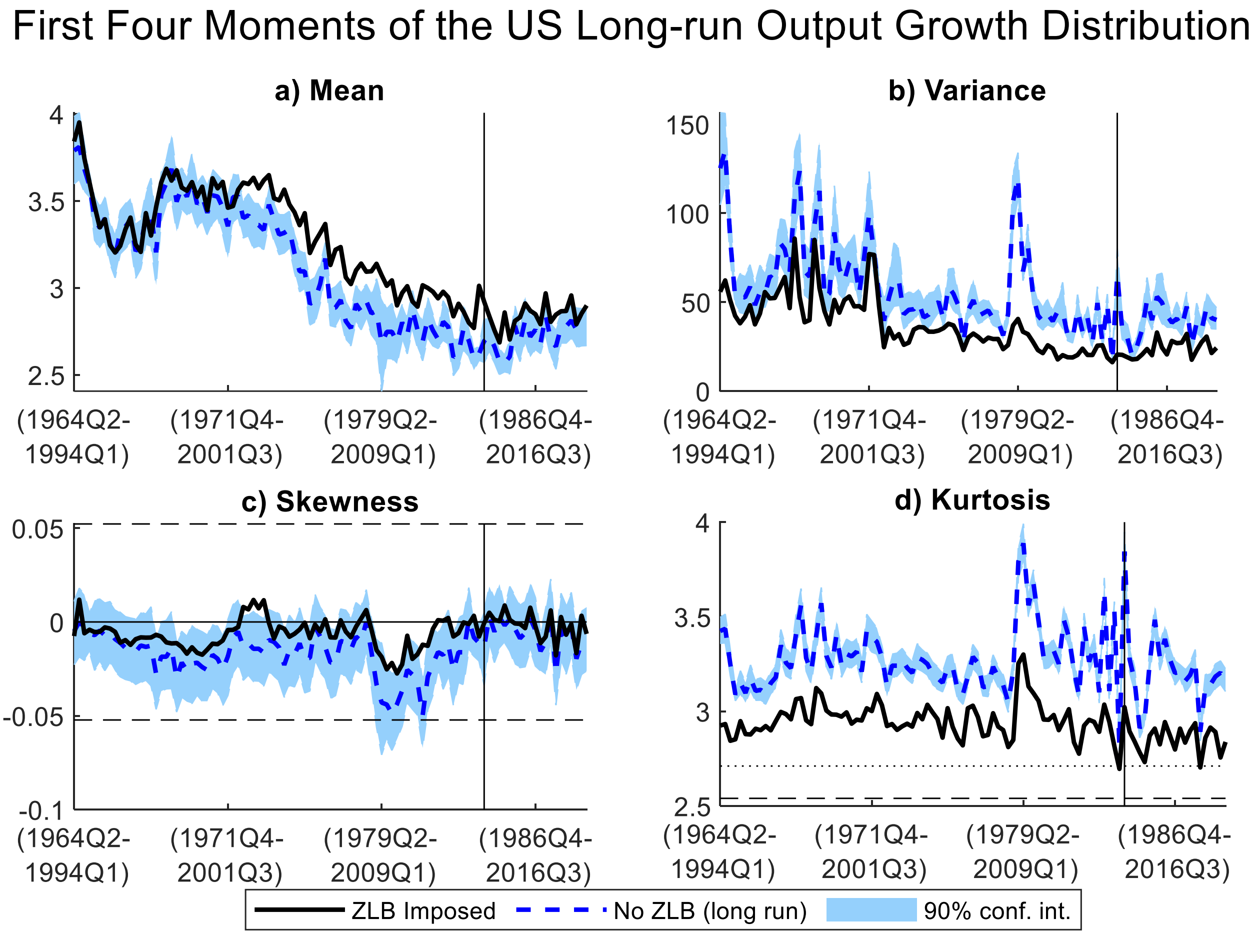
In particular, even without the ZLB the post-2014 reversal of the trend in the 10th percentile of the unconditional output growth distribution can still be observed (Figure C9, the right panel). Similarly, the change in unconditional skewness is still the most important development in the unconditional output growth distribution after the GFC enters the estimation windows (Figure C10, panel c).

Figure C9: The 10th Percentile of the Four-quarters-ahead (Left Panel) and Long-run (Right Panel) Output Growth Distribution from the Model Specification with and without Imposing the Zero Lower Bound



Note: The x-axis indicates the quarter for which the percentiles of conditional output growth distribution are estimated. The periods in parenthesis indicate the windows used to estimate the long-run output growth percentiles. The dashed lines indicate 90% confidence intervals.

Figure C10: Moments of the Long-run Output Growth Distribution of the Counterfactual with No Zero Lower Bound Imposed



Note: The x-axis indicates the range of the quarters of the estimation window. The solid vertical line indicates the estimation window 1984Q2-2014Q1, for which the risk profile changes its direction.

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ISSN 1803-7070